SUMS OF POSITIVE DENSITY SUBSETS OF THE PRIMES

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Given two subsets A and B of the primes, one can ask, in the spirit of the Goldbach conjecture, how large proportion of the natural numbers does the sum set $A+B=\{a+b : a\in A, b\in B\}$ cover. Answering this question, I will sketch a proof that if relative densities of A and B in the primes are α and β, then the density of $A+B$ in the natural numbers is at least $(1-o_{\alpha+\beta\to 0}(1))\alpha/(e^{\gamma}\log\log(1/\beta))$. This is the best possible lower bound as I will explain.