# ON POSITIVE DEFINITENESS OF MEET AND JOIN MATRICES

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Let $(P, \preceq)$ be a lattice and $f$ be a real-valued function on $P$. In addition, let $S = \{x_1, \ldots, x_n\}$ be a subset of $P$ which elements are distinct and arranged so that $x_i \preceq x_j \Rightarrow i \leq j$. The $n \times n$ matrix having $f(x_i \wedge x_j)$ as its $ij$ element is the *meet matrix* of the set $S$ with respect to $f$ and is denoted by $(S)_f$. Similarly, the $n \times n$ matrix having $f(x_i \vee x_j)$ as its $ij$ element is the *join matrix* of the set $S$ with respect to $f$ and is denoted by $[S]_f$. In case when $(P, \preceq) = (\mathbb{Z}_+, |)$ the matrices $(S)_f$ and $[S]_f$ are referred to as the GCD and LCM matrices of the set $S$ with respect to $f$.

In the literature there are several known sufficient conditions for the positive definiteness of GCD, LCM, meet and join matrices (see e.g. [2,3]). For example, it is known that if $(P, \preceq) = (\mathbb{Z}_+, |)$ and $(f * \mu)(d) > 0$ whenever $d \,|\, x_i$ for any $x_i \in S$, then the GCD matrix $(S)_f$ is positive definite (see [1, Theorem 1]).

Most of the existing results concerning positive definiteness of meet and join matrices are byproducts of certain factorization of these matrices. In this presentation we concentrate on the positive definiteness of these matrices and use a bit different approach as earlier. In case when the set $S$ is meet closed we give a sufficient and necessary condition for the positive definiteness of the matrix $(S)_f$. From this condition we obtain the known sufficient conditions as corollaries. We are also going to see that by making some additional assumptions about the set $S$ we are able to lighten the assumptions about the function $f$ still preserving the positive definiteness of the matrix $(S)_f$.

Dual theorems of these results for join matrices are presented as well. As examples we consider so called power GCD and power LCM matrices.

**References:**

[1] K. Bourque and S. Ligh, *Matrices associated with aritmetical functions*, Linear Multilinear Algebra 34 (1993) 261–267.

[2] I. Korkee and P. Haukkanen, *On meet and join matrices associated with incidence functions*, Linear Algebra Appl. 372 (2003) 127–153.

[3] B. V. Rajarama Bhat, *On greatest common divisor matrices and their applications*, Linear Algebra Appl. 118 (1989) 69–76.