On expressible sets for products

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Abstract

For a sequence of real numbers $\{a_n\}_{n=1}^{\infty}$ we call

$$E_\Pi\{a_n\}_{n=1}^{\infty} = \left\{ \prod_{n=1}^{\infty} \left(1 + \frac{1}{a_n c_n}\right) : c_n \in \mathbb{Z}^+ \right\}$$

its Π-expressible set. We calculate $E_\Pi\{a_n\}_{n=1}^{\infty}$ under various hypothesis on $\{a_n\}_{n=1}^{\infty}$. Where this is not possible we give some partial information on its contents. This investigation is a sequel to related investigations on the Σ-expressible sets of sums.

We say a sequence $\{a_n\}_{n=1}^{\infty}$ is Π-*irrational* if all the elements of the set $E_\Pi\{a_n\}_{n=1}^{\infty}$ are irrational numbers.

Hančl and Kolouch proved that if $\{a_n\}_{n=1}^{\infty}$ is a non-decreasing sequence of positive integers such that $\limsup_{n\to\infty} a_n^{\frac{1}{2^n}} = \infty$ and $a_n > n^{1+\varepsilon}$ where $\varepsilon > 0$ then the sequence $\{a_n\}_{n=1}^{\infty}$ is Π-irrational.

As an example of the type of result, we have the following.

Theorem *Let $\{a_n\}_{n=1}^{\infty}$ be a sequence of positive real numbers such that $\sum_{n=1}^{\infty} \frac{1}{2^n} \log a_n < \infty$. Then it is not Π-irrational.*

In this talk we will present more detailed results of sequence $\{a_n\}_{n=1}^{\infty}$ and its Π-expressible set.

This is joint work with Jaroslav Hančl and Radhakrishnan Nair.