Assignment for:
A Compressive Sensing Primer with Applications

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Version 1.0
August, 2014
Assignment
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1. Verify that $x = [e^{-\pi i/3} \ e^{\pi i/3} \ 0]^T$ is the unique minimizer of $P_1$ where
   
   $$A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}$$
   
2. Let $\Psi = \{\psi_1, \ldots, \psi_n\}$ be an orthonormal basis of $\mathcal{U}$. Let $h \in \mathcal{U}$, $h = \sum_{i=1}^{n} \alpha_i \psi_i$ be normalized such that $\|h\|_2 = 1$. Show that $h$ is maximally incoherent to the basis $\Psi$ if $\alpha_i = \alpha_j = c$, for all $i, j \in \{1, \ldots, n\}$.

3. Prove that the $n + 1$ vertices of a regular simplex in $\mathbb{R}^n$ centered at the origin form an equiangular tight frame.

4. The mutual coherence between two orthonormal bases can be defined as
   
   $$\mu(\Psi, \Phi) = \max_{i,j \in [n]} \ | < \psi_i, \phi_j > |$$
   
   Show that the following relation is valid:
   
   $$\frac{1}{\sqrt{n}} \leq \mu(\Psi, \Phi) \leq 1$$

5. Find a matrix $A \in \mathbb{R}^{2 \times 3}$ with minimal second-order restricted isometry constant.

The assignment is due on September 5th, 2014.