Coherent Tunneling of Cooper Pairs in Asymmetric Single-Cooper-Pair Transistors

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Abstract. We have calculated the $I-V$ characteristics of voltage biased asymmetric single-Cooper-pair transistors (SCPT), resulting from coherent Cooper pair tunneling across both the Josephson junctions (JJ), weak dissipation due the electromagnetic environment (EE) satisfying $\mathrm{Re}[Z(\omega)] \ll R_Q$, and quasiparticles. Due to the asymmetry, the smaller JJ is effectively probing the macroscopic quantum states of the island. A resonance occurs whenever the energy released in a tunneling of a single, or several, Cooper pair(s) across the smaller JJ matches to the energy needed to excite the island.

Keywords: Josephson junction, Single Cooper pair transistor, Cooper pair box, Coherent tunneling
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INTRODUCTION

Coherent tunneling of Cooper pairs across voltage biased SCPTs, disturbed by a dissipative environment, can lead to various phenomena [1]. A lot of research has been focused on charging effects of symmetric SCPTs, but also the $I-V$ characteristics across highly *asymmetric* SCPTs have been measured [2, 3], in order to probe the quantum states of mesoscopic JJs. In this paper we analyze the current across asymmetric SCPTs due to coherent Cooper pair tunneling, perturbed by an electromagnetic environment satisfying $\mathrm{Re}[Z(\omega)] \ll R_Q$, and quasiparticles. Instead to what has been done before, we assume a quantum coherence across both of the JJs and focus on the parameter range $E_{J1} \gg E_{J2} \sim E_c$ and the undergap region $V < 2\Delta_{\mathrm{gap}}/e$.

THE MODEL

We start the analysis from the Hamiltonian of the voltage biased SCPT, which is usually written as [1]

$$H_{\mathrm{SCPT}} = \frac{(Q_1 - Q_2 + Q_0')^2}{2C_\Sigma} - \frac{1}{2}(Q_1 + Q_2)V - E_{J1}\cos(\varphi_1) - E_{J2}\cos(\varphi_2). \tag{1}$$

The first term is the charging energy of the island, characterized by the energy $E_c = e^2/2C_\Sigma$. The second term gives the energy fed by the voltage source and the last two terms describe the Josephson currents across the two JJs. $Q_0' = C_gU + (C_1 - C_2)V/2$ is the polarisation charge due to the applied voltage V and a gate voltage U, $C_\Sigma = C_1 + C_2 + C_g$ where C_i is the capacitance of the i:th JJ, and C_g is the gate capacitance. The phase difference φ_i and the charge (gone through the junction i) Q_i are canonically conjugated variables.

For the case of asymmetric Josephson junctions ($E_{J1} \gg E_{J2}$), it is convenient to do a linear change of variables such that $Q = Q_1 - Q_2$, $Q_\Sigma = Q_2$, $\varphi = \varphi_1$ and $\varphi_\Sigma = \varphi_1 + \varphi_2$. The Hamiltonian (1) is now

$$H_{\mathrm{SCPT}} = \frac{(Q + Q_0)^2}{2C_\Sigma} - E_{J1}\cos(\varphi) - VQ_\Sigma - E_{J2}\cos(\varphi_\Sigma - \varphi). \tag{2}$$

Physically Q is the island charge, Q_Σ the charge tunneled across the small JJ (probe) and $Q_0 = C_gU - (C_2 + C_g/2)V$ is the new polarisation charge. We see that the first two terms describe the Hamiltonian of the Cooper pair box [3] (CPB), whereas the third term describes the charge gone through the probe. The last operator mixes these two subsystems weakly.

An EE satisfying $\mathrm{Re}[Z(\omega)] \ll R_Q$ perturbs the system by small voltage fluctuations V_f across the system, which can be included by a transformation $V \to V + V_f$ in the Hamiltonian (2). Also quasiparticles can tunnel, if an energy $2\Delta_{\mathrm{gap}}$ is released in a proper relaxation process. We calculate the resulting current due to these perturbations by using the golden rule, similarly as in Ref. [1].

RESULTS

At first, we neglect the quasiparticles, and analyze the effect of the dissipative EE. Typical $I-V$ characteristics obtained by numerical calculations are shown in Fig. 1. The nonresonant current, decreasing as a function of V, and the positions and the widths of the resonant peaks can be understood by analysing the eigenstates of the Hamiltonian (2) for $E_{J2} = 0$. These are $|\alpha, Q_0\rangle|n\rangle$, with the eigenenergies $E_{\alpha,Q_0,n} = E_{\alpha,Q_0} - 2eVn$, where $|\alpha, Q_0\rangle$ is the eigenstate of the CPB Hamiltonian with the quasicharge Q_0, eigenenergy E_{α,Q_0} and index $\alpha = 0, 1, 2\ldots$,

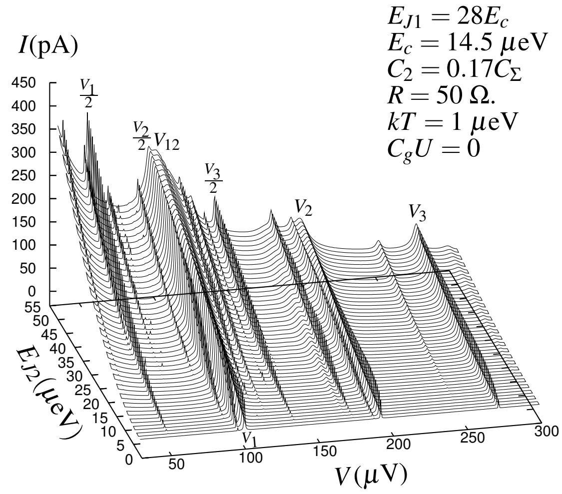

FIGURE 1. $I-V$ curves for different values of E_{J2}. No quasiparticles are taken into account. The first order resonances betwen the CPB ground $|0,Q_0\rangle$ and its excited states $|\alpha,Q_0\rangle$ are seen as current peaks at the voltages $V_1 \approx 100\ \mu$V, $V_2 \approx 190\ \mu$V and $V_3 \approx 270\ \mu$V. The resonance $|1\rangle \leftrightarrow |2\rangle$ (at V_{12}) is also seen nearby the voltage V_1. Higher order resonances (for example at $V_1/2$, $V_2/2$ $V_3/2$ etc.) become stronger for larger values of E_{J2}.

n tells the number of Cooper pairs tunneled across the probe. For small values of E_{J2} these states are very close to the correct eigenstates, expect for the degenerate situations $E_{\alpha,Q_0,n} \approx E_{\beta,Q_0,m}$, when the degenerate states (with the same quasicharge) are mixed srongly.

By taking the last term in the Hamiltonian (2) into account only perturbatively, one obtains for the nonresonant current an approximative result $I \sim 2(eE_{J2}/\hbar)^2 \mathrm{Re}[Z(2eV/\hbar)]/V$. A resonant current can flow in a situation, when the CPB ground and its excited state are mixed strongly, i. e. when $E_{0,Q_0,0} \approx E_{\alpha,Q_0,N}$. Resonances between two excited states do not usually produce $I-V$ peaks, since populations of them are small. Exceptions are the situations, when the ground state and two excited states are simultaneously in resonance.

By using a two-state approximation for a resonant situation between the states $|0,Q_0,0\rangle$ and $|\alpha,Q_0,1\rangle$ (a first order resonance) one gets the energy level splitting

$$\Delta E_{0,\alpha,Q_0} = \sqrt{(E_{J2}c_{0,\alpha,Q_0})^2 + (E_{\alpha,Q_0} - 2eV)^2}, \qquad (3)$$

where $c_{0,\alpha,Q_0} = |\langle\alpha,Q_0|\exp(i\varphi)|0,Q_0\rangle|$ and we have set that $E_{0,Q_0} = 0$. The strong mixing of states $|0,Q_0,0\rangle$ and $|\alpha,Q_0,1\rangle$ occurs when the first term inside the square root of eq. (3) is dominant. The corresponding $I-V$ peak has then a width $\Delta V \approx c_{0,\alpha,Q_0}E_{J2}/2e$. Higher order resonant situations ($N > 1$) will also produce $I-V$ peaks but with much smaller widths, since the states are usually connected by terms $\propto E_{J2}^N$.

A physical picture of a resonant situation is the following. Single, or several ($= N$), Cooper pairs tunnel *coherently* back and forth across the probe, and the CPB jumps simultaneously between the states $|0,Q_0\rangle$ and $|\alpha,Q_0\rangle$ cancelling the energy gain. Due to dissipative environment, this coherent process is from time to time interupted by an incoherent tunneling from the state $|\alpha,Q_0\rangle$ to some other state $|\beta,Q_0\rangle$. For $E_{J1} \gg E_c$ the fastest transition is $|\alpha,Q_0\rangle \to |\alpha-1,Q_0\rangle$, which then relaxes to $|\alpha-2,Q_0\rangle$ and so on, until the system is again in the ground state, expect that N Cooper pairs has tunneled across the probe. The slowest is the $|1,Q_0\rangle \to |0,Q_0\rangle$ transition, and therefore the maximum current is of the same order for every same order resonance. Furthermore, $I-V$ areas $A_{0\leftrightarrow\alpha,Q_0}$ of the first order resonances satisfy $A_{0\leftrightarrow\alpha,Q_0}/A_{0\leftrightarrow1,Q_0} \approx c_{0,\alpha,Q_0}/c_{0,1,Q_0}$, i. e. the ratio of the linewidths. For comparison, if the tunneling across the probe is only incoherent [2], one gets that the areas $\propto (c_{0,\alpha,Q_0}/c_{0,1,Q_0})^2$ drop faster with increasing α, and the linewidths increase $\propto \alpha$.

A new channel for the charge transport can open, whenever the energy $2\Delta_{\text{gap}}$ needed for a quasiparticle to tunnel, is released in a process. A quasiparticle tunneling across the probe (to the positive direction) is associated with an energy release eV and a quasicharge change $Q_0 \to Q_0 - e$, whereas the tunneling across the larger junction changes only Q_0 to $Q_0 + e$. A simultaneous quasiparticle tunneling across the probe and a transition $|\alpha,Q_0\rangle \to |\beta,Q_0-e\rangle$ is possible if $\delta E = E_{\alpha,Q_0} - E_{\beta,Q_0-e} + eV > 2\Delta_{\text{gap}}$ and occurs with a rate $\Gamma^{qp}_{\alpha\to\beta} \approx \delta E|\langle\beta,Q_0-e|\exp(-i\varphi/2)|\alpha,Q_0\rangle|^2/e^2R_2$, where R_2 is the normal state resistance of the probe. From this it follows, that the current in the resonant situations above $V > 2\Delta_{\text{gap}}/3e$ can be strongly enhanced due to quasiparticle tunneling.

In conclusion, Cooper pair tunneling across an asymmetric SCPT perturbed weakly by a dissipative EE, leads to resonant current peaks whenever the energy released in the tunneling of a single, or several, Cooper pair(s) across the probe equals the energy needed to excited the equivalent CPB circuit. Compared with the case of only incoherent tunneling across the probe, stronger resonances due to the higher excited states of the CPB are obtained. Quasiparticle tunneling can increase the tunneling rates in resonant situations above $V > 2\Delta_{\text{gap}}/3e$.

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