Klassinen kenttäteoria 763629S (3 ov, kevään 2005 kurssin mukaan)
Tentti 13.5.2005

1. Let us study the action integral of a free particle
$$S = -mc \int_a^b ds. \tag{1}$$
Derive from this the Lagrange function of a free particle, and show that it reduces to the well known expression in the nonrelativistic limit.

2. Justify that
$$\tilde{\rho} = \rho\sqrt{1-\frac{v^2}{c^2}} \tag{2}$$
is a four-scalar, where ρ is the charge density (charge per volume). Starting from the action
$$S = -\sum \int mcds - \sum \int eAds - \frac{1}{2c}\int \frac{\partial A}{\partial x_i}\frac{\partial A}{\partial x^i} d\Omega, \tag{3}$$
where the summation is over particles (charges e, masses m and line elements ds) and $d\Omega = dx^0dx^1dx^2dx^3$, derive the equation of motion for the scalar field $A(\boldsymbol{r}, t)$.

3. Let's study *charge conjugation*, where all charges change sign, $e \to -e$. Show that all Maxwell's equations and the equation of motion with Lorentz-force remain invariant when the fields are changes appropriately. What happens to the relation $\boldsymbol{j} = \sigma\boldsymbol{E}$?.

4. Write the components of the electromagnetic field tensor
$$F_{ki} = \frac{\partial A_i}{\partial x^k} - \frac{\partial A_k}{\partial x^i} \tag{4}$$
using electric and magnetic fields $\boldsymbol{E}$ and $\boldsymbol{B}$. What Maxwell equations (in 3-vector form) are given by the relation
$$\frac{\partial F_{ik}}{\partial x^l} + \frac{\partial F_{kl}}{\partial x^i} + \frac{\partial F_{li}}{\partial x^k} = 0. \tag{5}$$

5. Tell why the Coulomb's law
$$\boldsymbol{E}(\boldsymbol{r}) = \frac{1}{4\pi\epsilon_0}\frac{e(\boldsymbol{r}-\boldsymbol{r}_0)}{(\boldsymbol{r}-\boldsymbol{r}_0)^3}. \tag{6}$$
is not acceptable as a general relation in relativistic electrodynamics. Tell by words what types of relations one gets instead for the potentials ϕ and $\boldsymbol{A}$, and the fields $\boldsymbol{E}$ and $\boldsymbol{B}$ in the general case.

Fill in the course evaluation form.