Klassinen kenttäteoria 763629S (3 ov, kevään 2005 kurssin mukaan)
Tentti 10.6.2005

1. (a) Explain generally what is meant by a conservation law in field theory and what is its general mathematical form?

   (b) Explain by words what different parts there are in the action of the electrodynamics of particles?

2. Let us consider the Lagrangian

$$L = \int dV \mathcal{L}\left(u, \boldsymbol{\nabla} u, \frac{\partial u}{\partial t}; \boldsymbol{x}, t\right). \tag{1}$$

   Derive the Lagrange differential equations by varying the action. What kind of boundary conditions have to be assumed?

3. Write down the time component of the equation

$$mc\frac{du_k}{ds} = eF_{ki}u^i \tag{2}$$

   using three-vectors and scalars. Explain what it means physically. As a reminder

$$F_{ki} = \frac{\partial A_i}{\partial x^k} - \frac{\partial A_k}{\partial x^i}. \tag{3}$$

4. Let us consider two identical point charges with charge q separated by a distance a. Calculate the Coulomb repulsion between the charges using Maxwell's stress tensor acting through a surface separating the charges,

$$\sigma_{\alpha\beta} = \epsilon_0\left(\frac{1}{2}\delta_{\alpha\beta}E^2 - E_\alpha E_\beta\right) + \frac{1}{\mu_0}\left(\frac{1}{2}\delta_{\alpha\beta}B^2 - B_\alpha B_\beta\right). \tag{4}$$

5. Tell why the Coulomb's law

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{1}{4\pi\epsilon_0}\frac{e(\boldsymbol{r}-\boldsymbol{r}_0)}{(\boldsymbol{r}-\boldsymbol{r}_0)^3}. \tag{5}$$

   is not acceptable as a general relation in relativistic electrodynamics. Tell by words what types of relations one gets instead for the potentials ϕ and $\boldsymbol{A}$, and the fields $\boldsymbol{E}$ and $\boldsymbol{B}$ in the general case.

Fill in the course evaluation form.