5G Localization: Unlocking new Dimensions in Radio-based Positioning

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The 5G Positioning Challenge

Even when the line-of-sight is blocked?

And build a map of the environment?

From the signal received from a **single base station** in an **unknown propagation environment**, can we determine the **UE position, heading, clock bias**?
Outline

• Part I: Principles of radio-based positioning
• Part II: 5G positioning
  – 5G positioning: 5 selling points
  – 5G positioning in cmWave (below 6 GHz)
  – 5G positioning in mmWave (above 28 GHz)
  – 5G cooperative positioning
• Part III: 5G joint positioning and communication
• Conclusions
• References
Motivating questions

- What is radio-based positioning?
- How does it relate to other forms of positioning?
- What are the processes in radio-based positioning?
- Can you give examples or radio-based positioning?
- What is special about 5G in radio-based positioning?
- What are the important applications of 5G positioning?

Literature overview:
Outline

• Part I: Principles of radio-based positioning
• Part II: 5G positioning
  – 5G positioning: 5 selling points
  – 5G positioning in cmWave
  – 5G positioning in mmWave
  – 5G cooperative positioning
• Part III: 5G joint positioning and communication
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Principles: outline

- Measurements
- Performance bounds
- Algorithms
- Mobility
Main idea of radio-based positioning

- Waveform conveys information about geometry

- Different measurements can be taken
  - Signal strength
  - Time
  - Angle
  - (Doppler)
Example of time-based measurements
Signal strength

- **Principle**
  - Path loss equation: $P_r[\text{dBm}] = P_t[\text{dBm}] + K[\text{dB}] - 10\gamma \log_{10} \frac{d}{d_0}$
  - Learn parameters from data
  - Map received power to distance

- **Challenges**
  - Not one-to-one mapping
  - Many meters distance uncertainty
  - More common with fingerprinting

---

Path loss scatter plot for all the residential measurement data (not including data from tree lines).

---

Time: time of arrival (TOA)

Operation

Estimated in the clock of the receiver

\[ \hat{r} = \frac{d}{c} + B + n \]

Challenges

Clock bias must be removed before converting to distance
Obstacles: non-line-of-sight (NLOS)

Weakens LOS path, or can block completely (large positive range bias)
Induces extra delay \( L(\sqrt{\varepsilon_r} - 1)/c \) for size \( L \), relative permittivity \( \varepsilon_r \).
Time: two-way TOA

Operation

\[ \hat{\tau} = \frac{2d}{c} + n + \Delta + w \]

Challenges

- Estimated in the clock of the original transmitter
- Dedicated transaction per node pair
- Relies on dedicated hardware
Time: time difference of arrival (TDOA)

Operation

- Estimate $\hat{\tau}_i = d_i/c + B + n_i$
- Differential measurement $y_i = \hat{\tau}_i - \hat{\tau}_0, i > 0$ no longer depends on $B$
- One transmission per device

Challenges

- Requires tight synchronization among base stations
- Requires central processing unit
- Measurement noise of differential measurements is correlated
- Performance depends on choice of reference base station
Angle of arrival (AOA) and angle of departure (AOD)

Operation: AOA

\[ s(t - \tau)a(\theta) \]

OA Operation

- Observation depends on array response
- For ULA: \( a_k(\theta) = e^{j2\pi k \delta / \lambda \sin \theta}, k = 0, \ldots, N - 1 \)
- Should include additional unknown phase

AOD

\[ a^H(\theta)s(t - \tau) \]

Challenges

- Requires multiple antennas
- Antenna orientation must be known for measurement to be useful
Principles: outline

- Measurements
- Performance bounds
- Algorithms
- Mobility
Tool: Fisher Information and CRB

Problem: estimate deterministic unknown $\mathbf{x}$ from observation $\mathbf{z}$ given statistical model $p(\mathbf{z}|\mathbf{x})$

The Fisher information matrix (FIM):

$$
\mathbf{J}(\mathbf{x}) = \mathbb{E}_\mathbf{z}\{\nabla_\mathbf{x} \log p(\mathbf{z}|\mathbf{x}) \nabla_\mathbf{x}^T \log p(\mathbf{z}|\mathbf{x})\}
$$

measures “the amount of information the observation carries about the unknown”

FIM relates to estimation error covariance

$$
\mathbb{E}\{(\mathbf{x} - \hat{\mathbf{x}})(\mathbf{x} - \hat{\mathbf{x}})^T\} \succeq \mathbf{J}^{-1}(\mathbf{x})
$$

of any unbiased estimator $\hat{\mathbf{x}}(\mathbf{z})$

Cramér-Rao bound: lower bound on estimation error variance

$$
\mathbb{E}\{|\mathbf{x} - \hat{\mathbf{x}}|^2\} \geq \text{tr}(\mathbf{J}^{-1}(\mathbf{x}))
$$
FIM: more topics

- Equivalent FIM of a sub-vector:

\[
\begin{bmatrix}
A & B \\
B^T & C
\end{bmatrix}
\]

- Gaussian noise case is easier:

\[
z = m(x) + n, \quad n \sim \mathcal{CN}(0, \Sigma)
\]

\[
J(x) = \Re \left\{ \nabla_x m^H(x) \Sigma^{-1} \nabla_x m(x) \right\}
\]

- Transformation of variables: given injective mapping \( \eta = f(x) \)

\[
J(\eta) = TJ(x)T^T, \quad [T]_{i,j} = \partial \eta_i / \partial x_j
\]
Example

- **Problem** \( \mathbf{z} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \mathbf{x} + \mathbf{n}, \mathbf{n} \sim \mathcal{N}(\mathbf{0}, \sigma^2 \mathbf{I}) \)

\[
\eta = \begin{bmatrix} x_1 \\ \sin x_2 \end{bmatrix}
\]

- **FIM**

\[
\mathbf{J}(\mathbf{x}) = \frac{1}{\sigma^2} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \frac{1}{\sigma^2} \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}
\]

- **EFIM**

\[
\mathbf{J}^E(x_1) = \frac{1}{\sigma^2} (2 - 1 \times 1^{-1} \times 1) = \frac{1}{\sigma^2}
\]

\[
\mathbf{J}^E(x_2) = \frac{1}{\sigma^2} (1 - 1 \times 2^{-1} \times 1) = \frac{1}{2\sigma^2}
\]

- **Transformation**

\[
\mathbf{T} = \begin{bmatrix} 1 & 0 \\ 0 & \cos x_2 \end{bmatrix}
\]

\[
\mathbf{J}(\eta) = \mathbf{T} \mathbf{J}(\mathbf{x}) \mathbf{T}^T = \frac{1}{\sigma^2} \begin{bmatrix} 2 & \cos x_2 \\ \cos x_2 & \cos^2 x_2 \end{bmatrix} = \frac{1}{\sigma^2} \begin{bmatrix} 2 & \cos \arcsin \eta_2 \\ \cos \arcsin \eta_2 & 1 - \eta_2^2 \end{bmatrix}
\]

Depends on the value of the unknown.
Example: CRB on time delay estimation

- Model for OFDM with N subcarriers, unknown $\eta = [\tau \psi]^T$
  $$r = e^{j\psi}a(\tau) \circ s + n$$
  $$[a(\tau)]_k = \exp\left(-j2\pi \frac{k\tau}{NT_s}\right)$$

- Mean and derivative: $m(\tau, \psi) = e^{j\psi}a(\tau) \circ s$
  $$\nabla_{\eta} m(\theta, \psi) = [e^{j\psi}\dot{a}(\tau) \circ s \ j e^{j\psi}a(\tau) \circ s] \in \mathbb{C}^{N \times 2}$$
  $$[\dot{a}(\tau)]_k = -k\frac{j2\pi}{NT_s} [a(\tau)]_k$$

- FIM
  $$J(\eta) = \frac{1}{\sigma^2} \left[ \frac{4\pi}{(NT_s)^2} \sum_k k^2|s_k|^2 - \frac{2\pi}{NT_s} \sum_k k|s_k|^2 \frac{||s||^2}{||s||^2} \right]$$

- EFIM for delay
  $$J^E(\tau) = \frac{4\pi^2}{\sigma^2(NT_s)^2} \left[ \sum_k k^2|s_k|^2 - \frac{\left(\sum_k k|s_k|^2\right)^2}{||s||^2} \right]$$

- More bandwidth ($1/T_s$) is better, more energy in boundary subcarriers is better
Example: CRB on AOA estimation

- Model for AOA with unknown \( \eta = [\psi \theta]^T \) and \( N \) receive antennas
  \[
  r = e^{j\psi}a(\theta)s + n
  \]
  \[
  a(\theta) = [1 e^{j\pi \sin \theta} \ldots e^{j\pi(N-1) \sin \theta}]^T
  \]

- Mean and derivative: \( D(\theta) = \text{diag}(0, \pi \cos \theta, \ldots, \pi(N-1) \cos \theta) \)
  \[
  m(\psi, \theta) = e^{j\psi}a(\theta)s
  \]
  \[
  \nabla_\eta m(\psi, \theta) = [D(\theta)a(\theta)e^{j\psi}j s \ j e^{j\psi}a(\theta)s] \in \mathbb{C}^{N_r \times 2}
  \]

- FIM
  \[
  J(\eta) = \frac{|s|^2}{\sigma^2} \begin{bmatrix}
  \|D(\theta)a(\theta)\|^2 & \text{a}^H(\theta)D(\theta)a(\theta) \\
  \text{a}^H(\theta)D(\theta)a(\theta) & \|a(\theta)\|^2
  \end{bmatrix}
  \]
  \[
  = \frac{|s|^2 N}{\sigma^2} \begin{bmatrix}
  \pi^2(2N-1)(N-1)\cos^2 \theta & \pi(N-1) \cos \theta/2 \\
  \pi(N-1) \cos \theta/2 & 1
  \end{bmatrix}
  \]

- Equivalent FIM for AOA
  \[
  J^E(\theta) = \frac{|s|^2 N(N-1)(N+1)\pi^2 \cos^2(\theta)}{12\sigma^2}
  \]
  Nice scaling
Example: CRB on AOD estimation

- Model for AOD, with T transmissions, N transmit antennas, unknown $\eta = [\psi \theta]^T$
  $$ r^T = e^{j\psi} a^H(\theta) S + n^T \rightarrow r = e^{-j\psi} S^H a(\theta) + n $$

- What is the CRB?
- Can T be 1?
- How would you derive an estimator?
Example: CRB on AOD estimation

- Model for AOD, with T transmissions, N transmit antennas, unknown \( \eta = [\psi \theta]^T \)
  \[
  r^T = e^{j \psi} a^H(\theta) S + n^T \quad \Rightarrow \quad r = e^{-j \psi} S^H a(\theta) + n
  \]

- Mean and derivative: \( b(\theta) = S^H a(\theta) \)
  \[
  \nabla_\eta m(\theta, \psi) = \left[ e^{-j \psi} \dot{b}(\theta) - je^{-j \psi} b(\theta) \right] \in \mathbb{C}^{T \times 2}
  \]

- FIM:
  \[
  J(\eta) = \frac{1}{\sigma^2} \begin{bmatrix}
  \| \dot{b}(\theta) \|^2 & -\Re \{ b^H(\theta) \dot{b}(\theta) \} \\
  -\Re \{ b^H(\theta) \dot{b}(\theta) \} & \| b(\theta) \|^2
  \end{bmatrix}
  \]
  Large when signature varies across AOD
  Large when beaming in AOD

- EFIM for AOD
  \[
  J^E(\theta) = \frac{1}{2\sigma^2} \left( \| \dot{b}(\theta) \|^2 - \left( \frac{\Re \{ b^H(\theta) \dot{b}(\theta) \} \| b(\theta) \|^2}{\| b(\theta) \|^2} \right)^2 \right)
  \]

- T be at least two when both channel phase and gain are unknown
- Careful design of beams
Example: AOD estimation

• Using maximum likelihood

\[ r = \alpha S^H a(\theta) + n \]

\[ (\hat{\alpha}, \hat{\theta}) = \arg \min_{\alpha, \theta} \left\{ d(\alpha, \theta) \right\} \]

\[ \frac{\partial d}{\partial \alpha} = 0 \text{ implies} \]

\[ \hat{\alpha}(\theta) = \frac{a^H(\theta) S r}{\| S^H a(\theta) \|^2}, \]

which can be substituted so that

\[ \hat{\theta} = \arg \min \left( \frac{a^H(\theta) S r}{\| S^H a(\theta) \|^2}, \theta \right) \]
Example: estimating AOD and AOA

- Log likelihood $\log p(r|\theta)$ for 16 antennas ULA, $T=3$ transmissions, high SNR
- Prior valuable
CRB for positioning

- Generally on processed measurements (distance or angle)
- Sometimes based on waveform: “Direct positioning”
- Example: 1 agent ranging with M anchors
- Performance measure: position error bound (PEB) $P = \sqrt{\text{tr}(J^{-1}(x))}$
- FIM

\[
\begin{align*}
  z_m &= \|x - x_m\| + n_m \\
  \nabla_x \|x - x_m\| &= \nabla_x \sqrt{(x - x_m)^2 + (y - y_m)^2} \\
  &= \frac{1}{2\|x - x_m\|} \nabla_x (x - x_m)^2 + (y - y_m)^2 \\
  &= \frac{x - x_m}{\|x - x_m\|} \frac{1}{2\|x - x_m\|} \nabla_x (x - x_m)^2 + (y - y_m)^2
\end{align*}
\]

\[
J(x) = \sum_{m=1}^{M} \frac{1}{2\sigma^2} \frac{x - x_m}{\|x - x_m\|} \frac{1}{\|x - x_m\|} (x - x_m)^T
\]
Position error bound

- 3 anchors, visualize PEB $\mathcal{P} = \sqrt{\text{tr}(J^{-1}(x))}$ for different positions
Principles: outline

- Measurements
- Performance bounds
- Algorithms
- Mobility
Estimation basics

- Observation model \( \mathbf{r} = \mathbf{f}(\mathbf{x}) + \mathbf{n} \) with Gaussian noise
- Least squares (LS) estimator
  \[ \hat{\mathbf{x}} = \arg \min_{\mathbf{x}} \| \mathbf{r} - \mathbf{f}(\mathbf{x}) \|^2 \]
- Maximum likelihood (ML) estimator
  \[ \hat{\mathbf{x}} = \arg \max_{\mathbf{x}} p(\mathbf{r}|\mathbf{x}) \]
  \[ = \arg \max_{\mathbf{x}} - (\mathbf{r} - \mathbf{f}(\mathbf{x}))^T \mathbf{S}^{-1} (\mathbf{r} - \mathbf{f}(\mathbf{x})) \]
- Maximum a posteriori (MAP) estimator
  \[ \hat{\mathbf{x}} = \arg \max_{\mathbf{x}} p(\mathbf{x}|\mathbf{r}) \]
  \[ = \arg \max_{\mathbf{x}} - (\mathbf{r} - \mathbf{f}(\mathbf{x}))^T \mathbf{S}^{-1} (\mathbf{r} - \mathbf{f}(\mathbf{x})) + \log p(\mathbf{x}) \]
- Optimization generally not convex
Localization algorithms

- Typically layered

Waveform from source 1 → Estimate channel parameters → Convert to distance / bearing → Positioning unit → Position estimate

Waveform from source 2 → Estimate channel parameters → Convert to distance / bearing → Positioning unit → Position estimate

- Direct positioning

Waveform from source 1 → Estimate channel parameters → Convert to distance / bearing → Positioning unit → Position estimate

Waveform from source 2 → Estimate channel parameters → Convert to distance / bearing → Positioning unit → Position estimate

- Prior information provided in tracking mode (see later)
- Prior information can also be used for channel parameters
Example: ML positioning

- Well-behaved cost function
Example: ML positioning

• But relies on good anchor placement (see CRB)
Solving the LS problem

- Consider a system with one agent, TW-TOA. Measurements
  \[ r_k = \|x - x_k\| + n_k \]

- LS cost function
  \[ f_{LS}(x) = \sum_k (r_k - \|x - x_k\|)^2 \]

- Gradient descent:
  \[ \dot{x}^{(k)} = \dot{x}^{(k-1)} - \epsilon \nabla f_{LS}(\dot{x}^{(k-1)}) \]
  \[ \nabla f_{LS}(x) = \sum_k \nabla (r_k - \|x - x_k\|)^2 \]
  \[ = -2 \sum_k (r_k - \|x - x_k\|) \frac{x - x_k}{\|x - x_k\|} \]

- Needs an initial estimate
Point estimators through convexification

- LS reformulations
  \[
  \text{minimize}_{x,z,\varepsilon} \quad \|\varepsilon\|^2 \\
  \text{s.t.} \quad ||x - x_k|| = z_k \\
  \quad |z_k - r_k| \leq \varepsilon_k
  \]

- Relax circle to disk \( ||x - x_k|| \leq z_k \) leads to second order cone program

- Can be solved efficiently. Solution can be initial guess for gradient LS solver
Semidefinite programming relaxation

- LS reformulation

\[
\begin{align*}
\text{minimize} & \quad \sum_{k=1}^{N} \varepsilon_k \\
\text{s.t.} & \quad (\| \mathbf{x} - \mathbf{x}_k \| - r_k)^2 = \varepsilon_k
\end{align*}
\]

- Can be expressed as

\[
\begin{align*}
\text{minimize} & \quad \sum_{k=1}^{N} \varepsilon_k \\
\text{s.t.} & \quad (\mathbf{x}_k^T - 1) \begin{bmatrix} \mathbf{I}_2 & \mathbf{x} \\ \mathbf{x}^T & \mathbf{y} \end{bmatrix} \begin{bmatrix} \mathbf{x}_k \\ -1 \end{bmatrix} = v_k \\
v_k + r_k^2 - 2u_k r_k = \varepsilon_k \\
u_k^2 = v_k \\
y = \mathbf{x}^T \mathbf{x}
\end{align*}
\]

- SDP generally tighter but more complex

Not convex. Relax to SDP constraints

\[
v_k \geq u_k^2, \quad y \geq \mathbf{x}^T \mathbf{x}
\]

Positioning with belief propagation

- Suitable for distributed and large scale implementation
- Factorize joint distribution
  \[
p(x_1, x_2, x_a, x_b, r_{12}, r_{1a}, r_{1b}, r_{2a}, r_{2b}) = p(x_1)p(x_2)p(x_a)p(x_b) \\
\times p(r_{12}|x_1, x_2) \prod_{i \in \{a,b\}, j \in \{1,2\}} p(r_{ji}|x_i, x_j)
\]
- Create Bayesian graphical model and run message passing

Multiply independent sources of information
\[
n_i(x) \propto \psi_i(x, y) \prod_{j \in \Gamma(i)} m_j(x)
\]
Compute “extrinsic” marginals
\[
m_j(x) \propto \int_{x_i'} \psi_j(x_i', x_j) \psi_j(x_j, y) \prod_{k \in \Gamma(j) \setminus i} m_k(x_k) dx_i
\]
Principles: outline

- Measurements
- Performance bounds
- Algorithms
- Mobility
Dynamical model

- Discrete time evolution of the position
  \[ x_t = F_t x_{t-1} + B_t u_t + w_t, \quad w_t \sim N(0, Q_t) \]
  \[ y_t = h(x_t) + v_t, \quad v_t \sim N(0, R_t) \]

- Tracking algorithm maintains distribution of the state \( p(x_t|y_{1:t}) \)
- Form of distribution varies (e.g., Gaussian or samples)
- Optimal tracking for linear observation model: Kalman filter

- Nonlinear observation: extended Kalman filter, particle filter, etc

Extended Kalman filter

- Idea: Linearize observation
- Outcome at time t-1: \( p(x_{t-1}|y_{1:t-1}) \) of the form \( \mathcal{N}_{x_{t-1}}(\hat{x}_{t-1|t-1}, P_{t-1|t-1}) \)
- Prediction: \( p(x_t|y_{1:t-1}) \) of the form \( \mathcal{N}_{x_t}(\hat{x}_{t|t-1}, P_{t|t-1}) \)
  \[
  \hat{x}_{t|t-1} = F_t \hat{x}_{t-1|t-1} + B_t u_t \\
  P_{t|t-1} = F_t P_{t-1|t-1} F_t + Q_t
  \]
- Correction: linearize around predicted mean, approximately:
  \[
  h(x_t) = h(\hat{x}_{t|t-1}) + \nabla_{x_t} h(x_t)|_{x_t=\hat{x}_{t|t-1}} (x_t - \hat{x}_{t|t-1}) \]
- Then \( p(x_t|y_{1:t}) \) is of the form \( \mathcal{N}_{x_t}(\hat{x}_t|t, P_{t|t}) \)
  \[
  \hat{x}_{t|t} = \hat{x}_{t|t-1} + K_t(y_t - h(\hat{x}_{t|t-1})) \\
  P_{t|t} = (I - K_t H_t) P_{t|t-1} \\
  K_t = P_{t|t-1} H_t^T (H_t P_{t|t-1} H_t^T + R_t)^{-1}
  \]
- Works well in weak nonlinear regime
Particle filter (bootstrap filter)

- Idea: represent distributions by samples
- Outcome at time $t-1$: $p(x_{t-1}|y_{1:t-1})$ of the form $[x_{t-1}^{(1)}, \ldots, x_{t-1}^{(N_p)}]$ 
- Prediction: simulate the future $x_t^{(i)} \sim p(x_t|x_{t-1}^{(i)})$
- Correction: promote more likely samples $w^{(i)} \propto p(y_t|x_t^{(i)})$
- Resample: draw samples according to their weight $[x_t^{(1)}, \ldots, x_t^{(N_p)}]$
- Generally high complexity (large $N_p$, exponential in the dimension)

\[
\begin{align*}
\text{particle movement} & \quad \text{observation and resampling}
\end{align*}
\]
Distributed tracking with belief propagation

- **At time t-1:** local Gaussian distributions $\mathcal{N}(\hat{x}_{i,t-1|t-1}, P_{i,t-1|t-1})$

- **Prediction:** local update as in Kalman filter
  \[
  \hat{x}_{i,t|t-1} = F_{i,t} \hat{x}_{i,t-1|t-1} + B_{i,t} u_{i,t} \\
  P_{i,t|t-1} = F_{i,t} P_{i,t-1|t-1} F_{i,t}^T + Q_{i,t}
  \]

- **Correction:** account for measurements from references and neighbors using belief propagation

\[ q_i(x_i) \propto \psi_j(x_i, y) \prod_{j \in \Gamma(i)} m_{ji}(x_i) \]

\[ m_{ji}(x_i) \propto \int_{x_j} \psi_j(x_i, x_j) \psi_j(x_i, y) \prod_{k \in \Gamma(j) \backslash i} m_{kj}(x_i) \, dx_j \]
Summary

Models

- Radio signal convey position-related information
- Measurements can be related to distance and angle
- Time-based distance measurements require some form of synchronizations

Methods

- Bounds provide insight in performance
- Algorithms for static case: LS, ML, MAP, initialized by convexified problems
- Algorithms for mobile case: Bayesian filter
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Radio-based positioning

What do we mean by 5G?

- Large antenna arrays
- Directional transmission
- Large bandwidths
- Higher carriers
- Device-to-device communication
- Network densification
5 Selling points for 5G positioning

1. High carrier frequencies
2. Large bandwidths
3. Large number of antennas
4. D2D communication
5. Network densification
High carrier frequencies

- Received power due to path loss, shadowing, multipath fading
- Path loss: countered by array gains
- Shadowing: severe penetration loss so no shadowing
- Multipath fading: no diffraction, limited scattering and little reflection
- Communication channel is dominated by LOS and a few location-dependent clusters

Below 6 GHz: full matrix (i.i.d., Gaussian)
Above 28 GHz: low-rank matrix

\[ H(t) = \sum_{l=0}^{L-1} \alpha_l a(\theta_{rx,l}) a^H(\theta_{tx,l}) \delta(t - \tau_l) \]

Each “effective path” corresponds to cluster

Sparse communication channel, related to the physical environment
Large bandwidths

1. From Fisher information: large bandwidth leads to better delay (distance) estimation accuracy
2. More resolvable multipath components: two paths are resolvable when $|\tau_1 - \tau_2| \times B \gg 1$

High degree of resolvability of multipath

Large number of antennas

- From Fisher information
  - large number of RX antennas: better AOA resolvability
  - Large number of TX antennas: smaller beamwidth, better AOD resolvability
D2D communication

- 5G will have sidelinks
- Measurements *between* devices
- Can improve location accuracy and coverage

Cooperative positioning based on D2D measurements
Network densification

- Many access nodes
- High chance of LOS at short distances
- LOS link most useful for positioning

LOS link generally available for positioning

Table 3. Comparison of the LOS probability models for the UMi environment

<table>
<thead>
<tr>
<th>Model</th>
<th>$d_1$</th>
<th>$d_2$</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>3GPP UMi model ($d_1=18$, $d_2=36$)</td>
<td>18</td>
<td>36</td>
<td>0.023</td>
</tr>
<tr>
<td>Fitted ($d_1/d_2$) model ($d_1=20$, $d_2=39$)</td>
<td>20</td>
<td>39</td>
<td>0.001</td>
</tr>
<tr>
<td>NYU (squared) model ($d_1=22$, $d_2=100$)</td>
<td>22</td>
<td>100</td>
<td>0.026</td>
</tr>
</tbody>
</table>

Figure 12. UMi LOS probability for the three models considered.

Since the 3GPP 3D model [3GPP TR36.873] does not include an indoor scenario, and the indoor hotspot scenario in e.g. the IMT advanced model [ITU M.2135-1] differs from the office scenario considered in this white paper, an investigation on the LOS probability for indoor office has been conducted based on ray-tracing simulation. Different styles of indoor office environment were investigated, including open-plan office with cubical area, closed-plan office with corridor and meeting room, and also hybrid-plan office with both open and closed areas. It has been verified that the following model fits the propagation in indoor office environment best of the three models evaluated.

The verification results are shown in Table 4 and Figure 13. The LOS probability model used in ITU IMT-Advanced evaluation [ITU M.2135-1] and WINNER II [WINNER II D1.1.2] are also presented here for comparison. For the ITU and WINNER II model, parameterization results based on new data vary a lot from the original model. The results show that the new model has a good fit to the data in an average sense and can be used for 5G InH scenarios evaluation. However, note the high variability between different deployments and degrees of openness in the office area.
Outline

- Part I: Principles of radio-based positioning
- Part II: 5G positioning
  - 5G positioning: 5 selling points
  - 5G positioning in cmWave
  - 5G positioning in mmWave
  - 5G cooperative positioning
- Part III: 5G joint positioning and communication
- Conclusions
- References
5 Selling points for 5G positioning

1. High carrier frequencies
2. Large bandwidths
3. Large number of antennas
4. D2D communication
5. Network densification

Many possible paths, weaker connection to environment
5G positioning in cmWave: model

- Single antenna user device with unknown location and clock
- Multiple synchronized base stations with centralized processing
- Uplink pilot signal

\[ r(t) = \sum_{l=0}^{L-1} \alpha_l a(\theta_l) s(t - \tau_l) + n(t) \]

\[ \theta_0 = \arctan \left( \frac{x_{\text{user}} - x_{\text{BS}}}{y_{\text{user}} - y_{\text{BS}}} \right) \]

\[ \tau_0 = \| x_{\text{user}} - x_{\text{BS}} \| / c + \rho \]

- Many paths, so do we revert to \([h]_i \sim \mathcal{CN}(0, \sigma^2)\)?
- With large number of antennas: resolvable in angle
- With large bandwidth: resolvable in delay

Clock offset
Could be time-varying
5G positioning in cmWave: approaches

- **Model based**

- **Data driven**
Model-based cmWave 5G positioning for LOS

- Local processing at remote radio heads (RRH) with LOS for TOA and AOA (=DOA)
- Central processing for position and clock offset

After channel estimation, DoAs and ToAs are estimated and tracked at each LoS-RRHs using sequential estimation method, e.g., an Extended Kalman filter (EKF).

Position estimation
Position of the device is estimated and tracked within a central unit of a network by fusing the DoAs and ToAs from all the LoS-RRHs in a second EKF.
Model-based cmWave 5G positioning for LOS

- Random 3D trajectories, velocity 20-50 km/h
- METIS map-based ray-tracing channel modeling (including NLOS paths)
- AOA and TOA from two closest LOS base stations are fused
- 20 antennas
- Update interval: 100 ms
Model-based cmWave 5G positioning for NLOS

- Observation: \( \mathbf{r}(t) = \sum_{l=0}^{L-1} \alpha_l \mathbf{a}(\theta_l) s(t - \tau_l) + \mathbf{n}(t) \)
- Position consistency: The mobile is consistent with 4 paths (all LOS); The scatterer is consistent with 3 paths (all NLOS in this case)

\[
\begin{align*}
\text{convex problem:} & \quad \mathbf{X} \in \mathbb{C}^{G \times K} \\
\text{minimize} & \quad \|\mathbf{X}\|_{2,1} \\
\text{s.t.} & \quad \sum_{k=1}^{K} \left\| \mathbf{z}_k - \sum_{g=1}^{G} x_{gk} \mathbf{a}(\theta_k(\mathbf{p}_g)) \right\| \leq \varepsilon
\end{align*}
\]
Model-based cmWave 5G positioning for NLOS

- Search area: 100 m x 100m, 4 BS’s with 100 antennas each, Gaussian pulse emitted by source, 30 MHz baseband bandwidth at 7GHz carrier frequency, Simulated real indoor multipath channel. Proposed method “DiSouL”

- Baseline techniques:
  - Squared-Ranged Least-Squares (SR-LS) [Beck’08]
  - Instrumental Variables (IV) [Doğançay’03]
  - Stansfield estimator [Stansfield’46]
  - Direct Position Determination [Weiss’04]
Data-driven cmWave 5G positioning

- Idea: relation between channel at the BS and the user location
- Exploit sparsity

![Graph showing the magnitude of a sparse channel snapshot](image)

- Train neural network

Channel at BS  \[\rightarrow\] Input layer  \[\rightarrow\] Hidden layer  \[\rightarrow\] Output layer  \[\rightarrow\] UE location

Summary

• In cm-band: many multipath components, less bandwidth
• Large antenna arrays at base stations: use AOA, TOA, or entire response
• Model-based methods: Positioning requires BS cooperation
• Data-driven methods: Can be done with a single BS
Outline

- Part I: Principles of radio-based positioning
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5 Selling points for 5G positioning

1. High carrier frequencies
2. Large bandwidths
3. Large number of antennas
4. D2D communication
5. Network densification

Few paths, strong connection to environment
5G mmWave positioning: outline

• Positioning in the delay domain
  – Bounds
  – Algorithms

• Positioning in the delay and angle domain
  – Bounds
  – Algorithms

• Positioning in the angle domain
  – Bounds
  – Algorithms

Not covered in this tutorial
Problem and model

- Single anchor, multiple paths

\[ r(t) = \sum_{l=0}^{L} \alpha_l s(t - \tau_l) + n(t) \]

\[ \tau_0 = \| x_{BS} - x \| / c \]

\[ \tau_l = \| v_l - x \| / c \]

- Goal: determine user location, with unknown virtual anchors (possibly also unknown scatterers)
5G mmWave positioning: outline

- Positioning in the delay domain
  - Bounds
  - Algorithms
- Positioning in the delay and angle domain
  - Bounds
  - Algorithms
Performance bounds

- Fisher information matrix
  \[
  \eta = [\tau^T, \alpha^T]^T
  \]
  \[
  J(\eta) = \begin{bmatrix}
  J_{\tau,\tau} & J_{\tau,\alpha} \\
  J_{\tau,\alpha}^T & J_{\alpha,\alpha}
  \end{bmatrix}
  \]
  invert

- Remove nuisance parameter (EFIM)
  \[
  J(\tau) = J_{\tau,\tau} - J_{\tau,\alpha} J_{\alpha,\alpha}^{-1} J_{\tau,\alpha}^T
  \]
  invert

- Convert to position space: leads to singular FIM
  \[
  \tilde{\eta} = [x^T, v^T]^T
  \]
  \[
  J(\tilde{\eta}) = T J(\tau) T^T
  \]
  \[
  T = \nabla_{\tilde{\eta}} \tau(\tilde{\eta}) \in \mathbb{R}^{2L \times L}
Performance bounds

- If virtual anchors are known: nonsingular FIM
  \[ J(x) = T J(\tau) T^T \]
  \[ T = \nabla_x \tau(x) \in \mathbb{R}^{2 \times L} \]

- Multipath-assisted localization
- In practice user has unknown clock offset \( \rho \)
- When paths are all resolvable: \( |\tau_l - \tau_{l'}| B \gg 1, \forall l, l' \), then \( J(\tau) \) is diagonal, so each path provides independent information
Results

- 1 GHz bandwidth
- SNR at 1m LOS: around 30 dB
- PEB and uncertainty ellipses

Independent paths

[Diagrams showing position error bounds for independent and overlapping paths]

5G mmWave positioning: outline

- Positioning in the delay domain
  - Bounds
  - Algorithms

- Positioning in the delay and angle domain
  - Bounds
  - Algorithms
Algorithms

- If virtual anchors are known: nonsingular FIM
- If virtual anchors are unknown: needs tracking method with memory of virtual anchors locations
- SLAM: start from position with good prior and determine and track virtual anchors

![Diagram of Algorithms]

- MPC Estimation
- SINR Estimation
- Tracking Algorithm
- VA Discovery (SLAM)
- VA Memory
- Data Assoc.
- \( \mathcal{A}_n \)
- \( \mathcal{A}_{n, \text{ass}} \)
- \( \mathcal{A}_{n, \text{ass}} \)
- \( \{ \| \mathbf{a}_k - \hat{\mathbf{p}}_n \| \, | \mathbf{a}_k \in \mathcal{A}_n \} \)
Results @ 1 GHz

- Known virtual anchors

Results @ 1 GHz

- Unknown virtual anchors

Results @ 100 MHz

- Unknown virtual anchors

5G mmWave positioning: outline

- Positioning in the delay domain
  - Bounds
  - Algorithms

- Positioning in the delay and angle domain
  - Bounds
  - Algorithms
5G mmWave positioning: mathematical model

\[ H(t) = \sum_{l=0}^{L-1} h_l a_{rx}(\theta_{rx,l}) a_{tx}^H(\theta_{tx,l}) \delta(t - \tau_l) \]

Limited number of RF chains. Precoding matrix \( \mathbf{F} \) and combining matrix \( \mathbf{W} \).

\[ \mathbf{y}(t) = \mathbf{W}^H \sum_{l=0}^{L-1} \mathbf{H}_l \mathbf{F} x(t - \tau_l) + \mathbf{W}^H \mathbf{n}(t) \]

\[ \mathbf{H}_l = h_l a_{rx}(\theta_{rx,l}) a_{tx}^H(\theta_{tx,l}) \]

Estimate position (and orientation and clock bias) in the presence of unknown environment.
Some geometric intuition: downlink

- Abstraction
- Known BS position
- Synchronized

<table>
<thead>
<tr>
<th>Estimate</th>
<th>Position</th>
<th>Orientation</th>
</tr>
</thead>
<tbody>
<tr>
<td>AOA, AOD</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>AOA, TOA</td>
<td>no*</td>
<td>no</td>
</tr>
<tr>
<td>AOD, TOA</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>AOA, AOD, TOA</td>
<td>yes</td>
<td>yes</td>
</tr>
</tbody>
</table>

*unless orientation is known
Some geometric intuition: uplink

- Abstraction
- Known BS position
- Synchronized

**Table:**

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<td>no*</td>
<td>no</td>
</tr>
<tr>
<td>AOA, TOA</td>
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<td>no</td>
</tr>
<tr>
<td>AOA, AOD, TOA</td>
<td>yes</td>
<td>yes</td>
</tr>
</tbody>
</table>

*unless orientation is known
5G mmWave positioning: outline

- Positioning in the delay domain
  - Bounds
  - Algorithms
- Positioning in the delay and angle domain
  - Bounds
  - Algorithms
5G mmWave positioning: outline

- Positioning in the delay domain
  - Bounds
  - Algorithms
- Positioning in the delay and angle domain
  - Bounds
  - Fisher information
  - Uplink vs downlink
  - Performance examples
  - Synchronization
  - Algorithms
Performance bounds

- Received waveform

\[
y(t) = W^H \sum_{l=0}^{L-1} H_l F x(t - \tau_l) + W^H n(t)
\]

\[
H_l = h_l a_{rx}(\theta_{rx,l}) a_{tx}^H(\theta_{tx,l})
\]

- Channel parameters and location parameters

\[
\tau_0 = ||x||/c
\]
\[
\tau_l = ||x - v_l||/c, l > 0
\]
\[
\theta_{tx,0} = \arctan \left( \frac{y}{x} \right)
\]
\[
\theta_{tx,l} = \arctan \left( \frac{y - y_l}{x - x_l} \right)
\]
\[
\theta_{rx,0} = \arctan \left( \frac{y}{x} \right) + \pi - \alpha
\]
\[
\theta_{rx,l} = - \arctan \left( \frac{y - y_l}{x - x_l} \right) + \pi - \alpha
\]

- Virtual anchors have no physical meaning for scatterers

- Parametrization can also be in terms of scatterers and points of incidence
Fisher information matrix of channel parameters

- **Unknown parameter** \( \eta = [\theta_R^T, \theta_T^T, \tau^T, h^T]^T \)
- **Noise-free observation**
  \[
  m(t) = W^H \sum_{l=0}^{L-1} H_l F x(t - \tau_l)
  \]
- **FIM has entries**
  \[
  \Phi(x, x') = \frac{2}{N_0} \Re \left\{ \int \frac{\partial m(t)^H}{\partial x} \frac{\partial m(t)}{\partial x'} dt \right\}
  \]

This part was available in delay domain
Fisher information matrix of channel parameters

- Each sub-block (e.g., $J(\theta_R, \theta_R)$, $J(\theta_R, \tau)$) is Hadamard product

\[ \mathbf{J}(\theta_R, \theta_R), \mathbf{J}(\theta_R, \tau) \]

- Tends to diagonal when paths have distinct AOA (for large number of receive antenna)

- Tends to diagonal when paths have distinct AOD (for large number of transmit antennas), under full MIMO

- Tends to diagonal when paths have distinct delays (for large signal bandwidth)

- Conclusion: each sub-block will be almost diagonal
Fisher information matrix of channel parameters

- Original FIM

\[
\eta = [\theta_R^T, \theta_T^T, \tau^T, h^T]^T
\]

- Rearrange parameters

\[
\eta = [\eta_0^T, \eta_1^T, \ldots, \eta_{L-1}^T]^T \quad \eta_l^T = [\theta_{rx,l}, \theta_{tx,l}, \tau_l, h_l]
\]

Each path provides independent information
Fisher information matrix in position space

- Remove channel parameters with Schur complement
- Introduce parameter of interest $\tilde{\eta} = [x^T \alpha v_1^T \ldots v_{L-1}^T]^T$
- Determine FIM $J(\tilde{\eta}) = TJ(\eta)T^T$
- $T = \nabla \tilde{\eta}(\tilde{\eta}) \in \mathbb{R}^{(2L+1) \times 3L}$
- FIM can be nonsingular (Potential for snapshot SLAM)
- Compute EFIM $J^E(x, \alpha)$ of position and orientation
- From EFIM we compute PEB and OEB
- EFIM can be expressed as sum over the paths
5G mmWave positioning: outline

- Positioning in the delay domain
  - Bounds
  - Algorithms
- Positioning in the delay and angle domain
  - Bounds
    - Fisher information
    - Uplink vs downlink
    - Performance examples
    - Synchronization
  - Algorithms
**Uplink vs downlink**

**Downlink**
- Beam in *known* direction
- FIM in channel space
- Position relates to delay, AOD
- FIM in location space
- Leads to different scaling.

**Uplink**
- Beam in *unknown* direction
- FIM in channel space
- Position relates to delay, AOA
- FIM in location space
- Leads to different scaling.

Different!

Same!
Questions

• Is it better to do downlink or uplink positioning?
• Is it better to focus energy or provide more coverage?
• Is it better to send beams sequentially or in parallel?
• Is it beneficial to have more receive antennas?
• Is it beneficial to have more transmit antennas?
• Is it beneficial to have more bandwidth?
• Where should I place antennas? Transmitter or receiver?

To answer, we need:
• A reference scenario
• Performance metrics
• Quantities to be maintained
Reference scenario

- Transmitter at [0,0], receiver at [10, 2], each 32 antenna ULA, OFDM with 32 subcarriers, 100 MHz bandwidth, 30 GHz carrier
- TX has hybrid array, RX has fully digital array
- BS has orientation 0, UE has orientation $\pi/4$ (i.e., looking towards BS)
- Codebook:
  - DFT codebook centered around 0 direction
  - Directional codebook bounded to $-\pi/4, + \pi/4$
Reference with 20 beams

- Directional (left) vs DFT (right)
Metrics and preserved quantities

- Metrics: PEB and REB
- Preserved metrics: total energy during positioning
  - More beams: less energy per beam
  - More subcarriers: less energy per subcarrier (longer time)
More receiver antennas always helps REB and PEB: increased SNR
Sequential vs parallel transmission: similar performance (more noisy, not shown)
PEB: downlink PEB better for low $N_r$, since UL needs many Rx antennas to determine position via AOA (DL uses AOD)
similar REB, but for large $N_r$, AOD estimation is better in DL
Impact of UE orientation

- **PEB**: DL not affected, since AOD and delay estimation is always good. In UL, beams can point in wrong directions.
- **REB**: UL similar, but DL PEB also affected, since AOA estimation is not possible along the end-fire direction.
DFT codebook: impact of number of beams

- **PEB**: DL needs 6 beams for UE to be in coverage, UL needs 19. Performance degrades for more beams, as they spread energy in uninteresting directions. DL needs 2 beams, UL needs 1 (and is close to main lobe, with more beams reducing power before UE is fully illuminated).
- **REB**: UL needs many beams before SNR is high.
- Difference UL vs DL more pronounced with larger UE orientation.
DFT codebook: impact of transmit antennas

- More TX antennas leads to narrower beams, allows to focus on the user
- After 20 antennas, AOD range of 20 beams gets progressively smaller:
  - UL: BS is out of coverage in UL after 32 antennas
  - DL: UE is out of coverage in UL after 103 antennas
DFT codebook: fixed total number of antennas

- Significant impact of UE orientation in UL
- PEB in UL: better with more transmit antennas
- PEB in DL: better with more receive antennas
DFT codebook: \#beams = \#TX antennas

- Result for directional codebook is similar
- UL: more beams leads better coverage, but reduced power per beam
DFT codebook: impact of bandwidth

- Result for directional codebook is similar
- REB does not care about bandwidth
- PEB reduces with more bandwidth, but then becomes limited by AOA (UL) or AOD (DL) estimation
- Increasing bandwidth only useful is number of antennas is large enough
- Beamsquint was ignored here
Number of antennas vs inter-element spacing

- ULA with 10 vs 40 elements
- ULA with $\lambda/2$ spacing vs $2\lambda$ spacing

- Figure shows log-likelihood function for AOA estimation
- Both achieve same resolution, but larger spacing creates ambiguities
Summary

• 1 RF chain is enough, but we need multiple transmissions
• More RX antennas is always good
• More bandwidth is good, but only up to a point
• For joint position & orientation estimation: equal number of antennas at TX and RX
• Precoding and combining must be designed with care
• Questions
  – What about synchronization?
  – What about NLOS?
  – What when LOS is blocked?

*Take home message*
5G mmWave positioning: outline

- Positioning in the delay domain
  - Bounds
  - Algorithms
- Positioning in the delay and angle domain
  - Bounds
    - Fisher information
    - Uplink vs downlink
  - Performance examples
  - Synchronization
  - Algorithms
Results

- 3D scenario (unknown [position, azimuth, elevation])
- 12 x 12 arrays at TX and RX
- 38 GHz carrier, 125 MHz bandwidth (beam squint ignored)
- 1 mW transmit power, 16 training symbols
- Single path and 4 path channel
- No combining ($W = I$)

Single path (LOS) with 6 beams

Orientation error bound [deg]

Position error bound [m]
Four paths (including LOS) with 6 beams

Orientation error bound [deg]

Position error bound [m]

Scatterer locations
Single-base station positioning

Multipath helps!
Maps help
LOS is good, but not needed
Beamforming helps in the beams

Single-base station mapping

5G mmWave positioning: outline

- Positioning in the delay domain
  - Bounds
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- Positioning in the delay and angle domain
  - Bounds
  - Fisher information
  - Uplink vs downlink
  - Performance examples
  - Synchronization
  - Algorithms
Single-base station localization: clock offset

- Most existing studies ignore the tx-rx synchronization.
- Two alternatives to solve the clock offset:
  1. **The existence features in the environment** allows the clock synchronization using only one-way transmissions.
  2. **Two-way transmissions.**

Fisher Information Analysis

Sufficient conditions for localization and synchronization:
- LOS + 1 NLOS path (with or without map)
- 3 NLOS paths (2 NLOS with map)
Intuition: Visualization without LOS paths

- Each AOD determines a line from the BS
- Given a certain guess of the clock bias and orientation, each AOA determines a line from the user

Single-base station localization: clock offset

- Most existing studies ignore the tx-rx synchronization.
- Two alternatives to solve the clock offset:
  1. **The existence features in the environment** allows the clock synchronization using only one-way transmissions.
  2. **Two-way transmissions**.

Two-way single-base station localization

- Second alternative: use two-way transmissions

\[ \mathbf{F}_1, \mathbf{W}_1, \mathbf{s}_1(t), \mathbf{y}_1(t), \mathbf{n}_1(t) \]
\[ \theta_1, \phi_1, N_1, N_{B_1}, \mathbf{a}_1, \Delta_1 \]

No clock bias
- Transmit at \( t = 0 \)
- Receive at \( t = \tau^b \)

\[ \mathbf{F}_2, \mathbf{W}_2, \mathbf{s}_2(t), \mathbf{y}_2(t), \mathbf{n}_2(t) \]
\[ \theta_2, \phi_2, N_2, N_{B_2}, \mathbf{a}_2, \Delta_2 \]

Clock bias: \( B \)
- Transmit at \( t = t^b \)
- Receive at \( t = \tau^f \)
Single-base station mapping: clock offset

**Distributed Localization Protocol (DLP)**

\[
\begin{align*}
D_1 & \\
\tau_b &= 2\tau + e^f + \tau^D \\
y_1(t) & \\
\tau &= 0 \\
s_1(t) & \\
t & = 0 \\
\end{align*}
\]

\[
\begin{align*}
D_2 & \\
y_2(t) & \\
\tau^f &= \tau + B \\
\tau^D & \\
t^b &= \tau^f + \tau^D \\
s_2(t) & \\
t & = \tau \\
\end{align*}
\]

**Centralized Localization Protocol (CLP)**

\[
\begin{align*}
D_1 & \\
\tau_b &= \tau + t^b - B \\
y_1(t) & \\
\end{align*}
\]

\[
\begin{align*}
D_2 & \\
y_2(t) & \\
\tau^f &= \tau + B \\
t & = t^b \\
s_2(t) & \\
\end{align*}
\]
CDF of PEB with UE orientation angles of 0, and $N_{UE} = N_{BS} = 144$ antennas, $N_B = 25$ beams.
Single-base station mapping: clock offset

- UE and BS have a 12x12 array.
- 25 beams in both links providing SNR > 17 dB is 90% of locations.
- $f = 38$ GHz
5G mmWave positioning: outline

- Positioning in the delay domain
  - Bounds
  - Algorithms
- Positioning in the delay and angle domain
  - Bounds
  - Algorithms
Complete flowchart

- **PRS design**: beamformers and signal’s design.
  - Trade-off between accuracy and coverage
  - Trade-off between com and loc.
- Extend the position and mapping to more complex environment: clutter, reflecting surfaces, point scatters, hybrid & moving objects.

\[
y(t) = W^H \sum_{l=0}^{L-1} H_l F x(t - \tau_l) + W^H n(t)
\]

\[
H_l = h_l a_{rx}(\theta_{rx,l}) a_{tx}^H(\theta_{tx,l})
\]
Signal design

- **Question:** how should the BS signals be designed to allow for the best possible angle estimation?
- **Approach:** formulation convexified optimization problem

![Diagram](image)

N. Garcia, H. Wymeersch, D. Slock, "Optimal Precoders for Tracking the AoD and AoA of a mmWave Path", in *IEEE Transactions on Signal Processing*, 2018.
Complete flowchart

- PRS design: beamformers and signal’s design.
  - Trade-off between accuracy and coverage
  - Trade-off between com and loc.
- Extend the position and mapping to more complex environment: clutter, reflecting surfaces, point scatters, hybrid & moving objects.

\[ y(t) = W^H \sum_{l=0}^{L-1} H_l F x(t - \tau_l) + W^H n(t) \]
\[ H_l = h_l a_{rx}(\theta_{rx,l}) a_{tx}^H(\theta_{tx,l}) \]
Algorithms

- Observation model \( \mathbf{y}(t) = \mathbf{W}^H \sum_{l=0}^{L-1} \mathbf{H}_l \mathbf{F} \mathbf{x}(t - \tau_l) + \mathbf{W}^H \mathbf{n}(t) \)

  \[ \mathbf{H}_l = h_l \mathbf{a}_{\text{rx}}(\theta_{\text{rx},l}) \mathbf{a}^H_{\text{tx}}(\theta_{\text{tx},l}) \]

- We want to estimate TOA, AOA, AOD and L
- We know that each path provides independent information
- Use sparsity to extract channel parameters
- Idea: \( N_r \times D_t \rightarrow D_r \times 1 \)

  \[ \mathbf{a}(\theta_{\text{rx},l}) = \mathbf{U}_r^H \mathbf{a}_s \]

  Each column in \( \mathbf{U}_r \) is the response to a possible angle 1-sparse if (i) true angle is in dictionary and (ii) all columns are orthogonal

- Same for transmitter side

  \[ \mathbf{a}(\theta_{\text{rx},l}) \mathbf{a}^H(\theta_{\text{tx},l}) = \mathbf{U}_r^H \mathbf{a}_s \mathbf{b}_s^H \mathbf{U}_t \]

  \[ \sum_{l=0}^{L-1} \alpha_l \mathbf{a}(\theta_{\text{rx},l}) \mathbf{a}^H(\theta_{\text{tx},l}) = \mathbf{U}_r^H \mathbf{H}_s \mathbf{U}_t \]

  L-sparse
Choosing a good dictionary

- For ULA $a(\theta_{rx,l})_k = e^{j\pi k \sin \theta_{rx,l}}$, $k = 0, \ldots, N_r - 1$
- When angles take on value $\sin \theta_{rx,l} = 2l/N_r$
- DFT matrix is a reasonable dictionary

$$U_r = \left[ a\left(\frac{2(-N_r/2 + 1)}{N_r}\right), \ldots, a\left(\frac{2(N_r/2)}{N_r}\right)\right]$$

$$= \left[ a\left(-1 + \frac{2}{N_r}\right), \ldots, a(1)\right]$$

- In practice: angles not exactly on the DFT grid: approximately sparse
- Example: 64 antennas
Channel estimation algorithm

- OFDM signal on subcarrier $k$: $\mathbf{Y}[k] = \mathbf{W}^H \mathbf{U}_r^H \mathbf{H}_s[k] \mathbf{U}_t \mathbf{F} \mathbf{X}[k] + \mathbf{N}[k]$
- Vectorize
  $$\text{vec}(\mathbf{Y}[k]) = \mathbf{y}[k] = \mathbf{X}^T[k] \mathbf{F}^T \mathbf{U}_t^T \otimes \mathbf{W}^H \mathbf{U}_r^H \mathbf{h}_s[k] + \mathbf{n}[k]$$
- Introduce
  $$\mathbf{H}_s = [\mathbf{h}_s[0], \ldots, \mathbf{h}_s[N - 1]]$$
  $$\Psi[k] = \mathbf{X}^T[k] \mathbf{F}^T \mathbf{U}_t^T \otimes \mathbf{W}^H \mathbf{U}_r^H$$
- We can recover AOA / AOD by solving
  $$\minimize_{\mathbf{H}_s} \sum_{k=0}^{N-1} ||\mathbf{y}[k] - \Psi[k] \mathbf{h}_s[k]||^2 + \gamma ||\mathbf{H}_s||_{2,1}$$
- Can be solved with OMP
- Then recover gains (closed form) and delays (line search per path)
- Refine by adapting dictionary or post-processing (e.g., SAGE)
Example

- Matlab example

Matlab implementation available on https://github.com/henkwymeersch/5GPositioning
Complete flowchart

- PRS design: beamformers and signal’s design.
  - Trade-off between accuracy and coverage
  - Trade-off between com and loc.
- Extend the position and mapping to more complex environment: clutter, reflecting surfaces, point scatters, hybrid & moving objects.

\[
y(t) = W^H \sum_{l=0}^{L-1} H_l F x(t - \tau_l) + W^H n(t)
\]
\[
H_l = h_l a_{rx}(\theta_{rx,l}) a_{tx}^H(\theta_{tx,l})
\]
Geometric: from angles and delays to positions

- We have estimates of \( \eta = [\theta_R^T, \theta_T^T, \tau^T, h^T]^T \)
- Use transformation to \( \tilde{\eta} = [x^T \alpha^T v_1^T \ldots v_{L-1}^T]^T \)
- Apply Extended Invariance Principle (EXIP) and solve the following non-linear least square problem:

  \[
  \minimize_{\tilde{\eta}} \| \hat{\eta} - \eta(\tilde{\eta}) \|^2
  \]

Initialize by:

1. **When LOS path exists**: use (AOD,delay) of path with shortest delay to recover position, then (AOD, AOA) to recover orientation. Then virtual anchors (or scatterer locations) are easily recovered path by path.

2. **When LOS path does not exist**: try all possible orientations: each (AOD, AOA, delay) gives rise to a path. Intersection of two paths is position. Evaluate cost for each guess. 3 paths are needed in total.

3. **When not know if LOS exists**: try (1) and (2) and evaluate cost.
Results: 4 path channel (including LOS)

A. Shahmansoori, G.E. Garcia, G. Destino, G. Seco-Granados, H. Wymeersch
Results: 3 path channel without LOS

A. Shahmansoori, G.E. Garcia, G. Destino, G. Seco-Granados, H. Wymeersch
From angles & delays to positions: Bayesian approach

• Factor graph. The objective is to obtain the posterior distribution of the position parameters given the channel parameters.

\[ p(x_{\text{UE}}, \alpha_{\text{UE}}, B, x_{\text{VA},1}, \ldots, x_{\text{VA},L-1}|Z) \]

\[ = p(x_{\text{UE}})p(\alpha_{\text{UE}})p(B) \prod_{l=1}^{L-1} p(x_{\text{VA},l}) \]

\[ \times p(z_0|x_{\text{UE}}, \alpha_{\text{UE}}, B) \prod_{l=1}^{L-1} p(z_l|x_{\text{UE}}, \alpha_{\text{UE}}, x_{\text{VA},l}, B) \]

• Perform belief propagation
• Needs schedule
• Allows the introduction of prior information
• Facilitates tracking and hybridization with other sensors
Quick convergence, unknown bias leads to 1 meter penalty. Unknown orientation incurs no penalty.
Quick convergence, unknown bias leads to 1 meter penalty. Unknown orientation incurs no penalty.
Orientation estimation performance

Performance does not depend on prior of bias or orientation
Small difference between VAs with concentrated and vague prior.
Knowledge of bias is useful.
**Complete flowchart**

- **PRS design**: beamformers and signal’s design.
  - Trade-off between accuracy and coverage
  - Trade-off between com and loc.
- Extend the position and mapping to more complex environment: clutter, reflecting surfaces, point scatters, hybrid & moving objects.

Mathematical expressions:

\[ y(t) = W^H \sum_{l=0}^{L-1} H_l F_x(t - \tau_l) + W^H n(t) \]

\[ H_l = h_l a_{rx}(\theta_{rx,l}) a_{tx}^H(\theta_{tx,l}) \]
Data association: classical approach

- **Question**: how can the channel estimates be associated to prior map information?
- **Approach**: Compute expected likelihood ($S_{l,m}$: path $l$, map entry $m$, with 1 new map entry per path), find best global association ($x$). Hard decision works when sources are well separated, no clutter

\[
\begin{align*}
\text{maximize} & \quad \sum_{l=0}^{L-1} \sum_{m=1}^{M+L} x_{l,m} \log S_{l,m} \\
\text{s.t.} & \quad x_{l,m} \in \{0, 1\}, \quad \forall l, m, \\
& \quad \sum_{m=1}^{M+L} x_{l,m} = 1, \quad \forall l, \\
& \quad \sum_{l=0}^{L-1} x_{l,m} \leq 1, \quad \forall m,
\end{align*}
\]

Hyowon Kim, H. Wymeersch, Nil Garcia, G. Seco-Granados, Sunwoo Kim, "5G mmWave Vehicular Tracking", *Asilomar Conference on Signals, Systems, and Computers*, Oct 2018,
Classic approach: main idea

- Consider map with 3 entries
- We see three paths

<table>
<thead>
<tr>
<th>BS</th>
<th>Wall 1</th>
<th>Wall 2</th>
<th>New 1</th>
<th>New 2</th>
<th>New 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z_1$</td>
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<td>-6</td>
<td>-4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$z_2$</td>
<td>5</td>
<td>9</td>
<td>-3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$z_3$</td>
<td>-3</td>
<td>-2</td>
<td>-4</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

- Hard decisions!
- How about soft decisions?
Data association: factor graph approach

- Introduce data association (DA) variables (a,b)
- Create factor graph with vehicle state, VA states, DA variables


© Henk Wymeersch, Gonzalo Seco-Granados, 2017-2019

Frohle, Markus et al “Cooperative localization of vehicles without inter-vehicle measurements.” *WCNC, 2018.*
Data association: random finite set (RFS) approach

- Represent the number of VAs as a random set instead of a random vector
  \[ \mathcal{M} = \emptyset \] (no features present)
  \[ \mathcal{M} = \{m^1\} \] (one feature with state \(m^1\) present)
  \[ \mathcal{M} = \{m^1, m^2\} \] (two features \(m^1 \neq m^2\) present)
  \[ \vdots \]
  \[ \mathcal{M} = \{m^1, \ldots, m^m\} \] (\(m\) features \(m^1 \neq \cdots \neq m^m\) present).

- The probability hypothesis density (PHD) can approximate the RFS density (think mean and variance for vectors)
- Typical approach: particles for vehicle state, PHD for map conditioned on vehicle state

\[ X = \text{true location} \]
\[ \text{Lines} = \text{measurements} \]
Summary

- In mm-band: few multipath components, lots of bandwidth
- Paths resolvable in delay or angle: use AOA, DOA and TOA
- In delay domain:
  - Static: multipath positioning
  - Dynamic: simultaneous localization and mapping (SLAM)
- In angle & delay domain:
  - Static: multipath position and orientation estimation with single anchor
  - Dynamic: SLAM also possible
  - Beamforming plays an important role
- Questions:
  - What about angle and delay spread?
  - What about real measurements?
  - What about mobility? Multi-user positioning?
Outline

- Part I: Principles of radio-based positioning
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  - 5G positioning: 5 selling points
  - 5G positioning in cmWave
  - 5G positioning in mmWave
  - 5G cooperative positioning
- Part III: 5G joint positioning and communication
- Conclusions
- References
5 Selling points for 5G positioning

1. High carrier frequencies
2. Large bandwidths
3. Large number of antennas
4. D2D communication
5. Network densification
Cooperative positioning: idea

Improved accuracy

Improved coverage

What is benefit of measurement with both TX and RX have unknown position?
Cooperative positioning: outline

- Performance bounds
- Algorithms
Cooperative positioning model

- Network with N agent nodes ($I_a$), M reference stations ($I_r$)
- Graph with vertex set $\mathcal{V}$ and edge set $\mathcal{E}$
- Measurements $z_{ij} = f(x_i, x_j) + n_{ij}, (i, j) \in \mathcal{E}$
- Many possibilities
  - Sampled waveform
  - Distance, AOA, AOD estimate
  - Relative position
- Highest level of abstraction: $z_{ij} = x_i - x_j + n_{ij}$

More local processing

Statistics from 5G positioning algorithm or from PEB
Cooperative positioning: Fisher information

- **Unknowns**: $\eta = [x_1^T, \ldots, x_N^T]^T$
- **Observations**: $z_{ij} = x_i - x_j + n_{ij}$, $n_{ij} \sim \mathcal{N}(0, \Sigma_{ij})$
- **Fisher information matrix**

\[
J(\eta) = \\
\begin{cases} 
0 & (i, j) \notin \mathcal{E} \\
-\Sigma_{ij}^{-1} & (i, j) \in \mathcal{E}
\end{cases}
\]

\[
J(x_i, x_j) = \sum_{k \in N_i} \nabla_{x_i} (x_i - x_k) \Sigma_{ik}^{-1} \nabla_{x_i}^T (x_i - x_k)
\]

\[
= \sum_{j \in N_i} \Sigma_{ij}^{-1}
\]

\[
= \sum_{j \in N_i \cap \mathcal{I}_a} \Sigma_{ij}^{-1} + \sum_{j \in N_i \cap \mathcal{I}_r} \Sigma_{ij}^{-1}
\]
Cooperative positioning: Fisher information

- Fisher information matrix $J(\eta) = J_{\text{ref}} + J_{\text{coop}} - C$

  - Block-diagonal from references
  - Block-diagonal from other agents
  - Reduction of information since other agents have unknown position

- Example (for equal covariance per link)

  $$\Sigma J(x_1, x_2, x_3) = \begin{bmatrix} 2I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 2I \end{bmatrix} - \begin{bmatrix} 0 & 0 & I \\ 0 & 0 & I \\ I & I & 0 \end{bmatrix} = \begin{bmatrix} 3I & 0 & -I \\ 0 & 2I & -I \\ -I & -I & 2I \end{bmatrix}$$

  - Singular without cooperation
  - Full rank with cooperation

- $\text{PEB}(x_1) < \text{PEB}(x_2) < \text{PEB}(x_3)$
- FIM structure reminiscent of graph Laplacian
Cooperative positioning: outline

- Performance bounds
- Algorithms
Algorithms

Static scenario
- cooperative LS/ML/MAP with steepest descent solver
  \[
  \hat{x}^{(k)} = \hat{x}^{(k-1)} - \epsilon \nabla f_{LS}(\hat{x}^{(k-1)})
  \]
  \[
  \nabla f_{LS}(x) = \sum_{k} \nabla (r_k - \|x - x_k\|)^2
  \]
  \[
  = -2 \sum_{k} (r_k - \|x - x_k\|) \frac{x - x_k}{\|x - x_k\|}
  \]
- Can be initialized with SDP or SOCP solution
- Can be distributed

Dynamic scenario
- Extended Kalman filter on super-state
- Particle filter on super-state
- Can be distributed

Distributed tracking

- **At time t-1**: local Gaussian distributions $\mathcal{N}(\hat{x}_{i,t-1|t-1}, P_{i,t-1|t-1})$

- **Prediction**: local update as in Kalman filter
  
  $\dot{x}_{i,t|t-1} = F_{i,t} \dot{x}_{i,t-1|t-1} + B_{i,t} u_{i,t}$
  
  $P_{i,t|t-1} = F_{i,t} P_{i,t-1|t-1} F_{i,t}^T + Q_{i,t}$

- **Correction**: account for measurements from references and neighbors using belief propagation

\[ q_i(x_i) \propto \psi_i(x_i, y) \prod_{j \in \Gamma(i)} m_j(x_i) \]

\[ m_j(x_i) \propto \int_{x_i} \psi_j(x_i, x_j) \psi_j(x_j, y) \prod_{k \in \Gamma(j) \setminus i} m_k(x_j) \, dx_j \]
Example

- 100 mobile agents with low or high mobility

![Graph showing CCDF vs Localization error [m] for different conditions: Non-coop with \( \sigma_{\text{mob}} = 10 \) and \( \sigma_{\text{mob}} = 1 \), and Coop with \( \sigma_{\text{mob}} = 10 \) and \( \sigma_{\text{mob}} = 1 \). The x100 gain of cooperation is highlighted.]

Summary

- 5G can harness D2D communication
- Extra information to improve accuracy
- Distributed algorithms for cooperative tracking
- Extensions for SLAM
- Synchronization challenges
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Position information in the initial access (IA) problem

- Narrow beams: high SNR but where to point?

- Use out-of-band technology to provide position information. Use this information to improve IA.

- We now know: in-band can also be used
Benefit of out of band location information

- Location information provides angle information

![Diagram showing channel estimation procedure and channel estimation procedure are of 5 antennas and a single RF chain. The parameters of the channel needs to be sounded in fewer directions. In the case of high precision, the channel estimation duration. Such reduction is larger as the distance increases.](image)

Fig. 6 plots the expected receive SNR after performing channel estimation when location information is available to estimate the channel gain.

Distance [m]

<table>
<thead>
<tr>
<th>AOD = 20º and 70º, no prior knowledge</th>
<th>AOD = 20º, GNSS precision</th>
<th>AOD = 20º, high precision</th>
<th>AOD = 70º, GNSS precision</th>
<th>AOD = 70º, high precision</th>
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</thead>
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<tr>
<td>200</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Receive SNR [dB]

Fig. 6. Expected receive SNR versus distance.

Distance [m]

- Nil Garcia, Henk Wymeersch, Erik G. Ström, and Dirk Slock,
Benefit of in-band position information

Conventional hierarchical beam search
1. Transmit with broad beams
2. Feed back best beam
3. Refine beams and go to step 1

Modified hierarchical beam search
1. Transmit with broad beams
2. Feed back best beam and position information
3. Refine beams and go to step 1

Rate / PEB tradeoff

- **Model for communication**

\[
T_i \quad T_f \quad T_d
\]

- **Effective data rate**

\[
R = \left(1 - \frac{NMT_s}{T_f}\right) \log_2 \left(1 + \frac{|h|^2 P_{tx} S(w^*, f^*, \theta, \phi)}{\sigma^2}\right)
\]

\[
[w^*, f^*] = \arg \max_{w \in \mathcal{W}, f \in \mathcal{F}} \frac{|h|^2 P_{tx} S(w, f, \theta, \phi)}{\sigma^2}.
\]

- **Position information**

\[
\text{PEB} = \sqrt{\text{trace} \left( [J_\xi^{-1}]_{1:2,1:2} \right)},
\]

\[
J_\xi = \sum_{w \in \mathcal{W}, f \in \mathcal{F}} J_\xi(f, w),
\]

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- References
Conclusions

- Radio signals can provide location information
- 5G has important advantages here
- Harness resolvability in time, angle for
  - Single anchor localization
  - Tracking
  - SLAM
  - Location-aided communication
  - Radar
- Processing at BS or UE
5G positioning vs automotive FMCW radar

**5G positioning**
- Carrier above 28 GHz
- Large bandwidth (100+ MHz)
- Large antenna arrays (100+)
- Full-band ADC (100+MHz)
- Coordinated transmission
- Mainly communication
- Possible for positioning & mapping!

**FMCW radar**
- Carrier above 28 GHz
- Very large bandwidth (0.5-4 GHz)
- Multi-antenna arrays (2-4)
- Low-rate ADC (10-40 MHz)
- Uncoordinated transmission
- Mainly mapping
- Possible for communication?

[Diagram showing radar systems and applications.]
Challenges

- Good geometric mmWave channel models for positioning, including blockage, clustering and distributed sources
- Database of location-based channel measurements
- Design of precoding and combining for positioning, mapping
- Pilot design for positioning, mapping
- Fast algorithms for positioning, tracking, mapping
- Online synchronization for positioning
- Multi-user positioning, resource allocation for MU positioning
- Calibration of references (location, time)
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Basics of radio-based positioning

5G properties for positioning
References

Single anchor positioning

Cm-wave positioning

Mm-wave positioning in delay domain
Mm-wave positioning in delay and angle domain

References

Cooperative positioning


References

Location-aided communication

BS
\[ y \]
\[ \text{downlink} \]
\[ \theta_{\text{BS}_0} \]
\[ \tau_0 \]

\[ x \]

\[ \alpha \]

\[ \theta_{\text{UE}_0} \]

UE
Exercise 1 – Estimation of DOD

- The received signal consists of the superposition of $M_{BS}$ signals sent through the corresponding beams.
- You can assume that the signals are orthogonal, because they are sent either on different subcarriers or at different time slots, or they are orthogonal by design.
- The received signal can be expressed as:

$$y(t) = h a^H(\theta_{tx}) F x(t) + n(t)$$
$$= h \left( a^H(\theta_{tx}) f_1 x_1(t) + \cdots + a^H(\theta_{tx}) f_{M_{BS}} x_{M_{BS}}(t) \right) + n(t)$$
Exercise 1 – Estimation of DOD

Questions:

• Propose a method to estimate $\theta_{BS}$.

• How many beams $M_{BS}$ are needed for the problem to be identifiable?

• Write the signal model in the uplink for the same scenario (array at the BS and one antenna at the UE). Compare the position of the steering vector $\mathbf{a}(\theta_{BS})$ in the uplink and downlink signal models.

• Discuss how the number of antennas $N_{BS}$ will affect the estimation of the $\theta_{BS}$ in the uplink and in the downlink. In which case do you expect a more marked improvement with $N_{BS}$?
Propose a method to estimate $\theta_{BS}$.

- The signal $y(t)$ is passed through matched filters for each of the pilot signals. Assuming, without loss of generality, that the pilot signals have unit energy, the result is:

$$y = h \left[ a^H(\theta_{tx}) f_1 \right] + n \left[ \vdots \right] + h \left[ a^H(\theta_{tx}) f_{MBS} \right]$$

- We can use the least-square error method

$$\min_{h, \theta_{tx}} \left\| y - h \left[ a^H(\theta_{tx}) f_1 \right] \right\|^2 \rightarrow \max_{\theta_{tx}}\left\| \frac{y^H \left[ a^H(\theta_{tx}) f_1 \right]}{\left\| \frac{a^H(\theta_{tx}) f_{MBS}}{a^H(\theta_{tx}) f_{MBS}} \right\|^2} \right\|^2$$
Exercise 1 – Estimation of DOD - solution

• The cost function can be written as:

\[
\max_{\theta_{tx}} \frac{|a(\theta_{tx})^H F y^*|^2}{\|a(\theta_{tx})^H F\|^2}
\]

where \( F = [f_1 \ldots f_{MB_S}] \)

• We need \( MB_S \geq 2 \) for the problem to be identifiable, otherwise the cost function is constant and does not depend on \( \theta_{tx} \).

• At least two beams are needed to distinguish the changes in amplitude due to \( h \) from those due to the gain of the beams \( a(\theta_{tx})f_i \).

• The cost function can be seen as the projection of vector \( y^* \) on the subspace spanned by \( F^H a(\theta_{tx}) \). The dimension the subspace has to be greater than or equal to 2.
Exercise 1 – Estimation of DOD - solution

Write the signal model in the uplink for the same scenario (array at the BS and one antenna at the UE). Compare the position of the steering vector \( \mathbf{a}(\theta_{BS}) \) in the uplink and downlink signal models.

Discuss how the number of antennas \( N_{BS} \) will affect the estimation of the \( \theta_{BS} \) in the uplink and in the downlink. In which case do you expect a more marked improvement with \( N_{BS} \)?

\[
\mathbf{y} = h \mathbf{a}(\theta_{tx}) + \mathbf{n}
\]

- The dimension of the observation is the number of antennas \( N_{BS} \).
- The steering vector \( \mathbf{a}(\theta_{tx}) \) is directly observed, not through its product times \( \mathbf{F} \).
- Increasing the number of antennas \( N_{BS} \) causes:
  - An increase in the captured energy (array gain); effect \( \sim N_{BS} \)
  - Higher aperture to estimate \( \theta_{tx} \); effect \( \sim N_{BS}^3 \)
  - No penalty in power splitting between beams.
Exercise 2 – Position determination with clock offset

• We consider a 2D scenario with LOS and one NLOS produced by a point scatterer in unknown position $x_{s_1}$.

• The objective is to determine the position and orientation of the UE, $x_{UE}$ and $\alpha$, and possibly the scatterer from different set of channel parameters.
Exercise 2 – Position determination with clock offset

1. Assume only LOS
   • Assume that BS and UE have synchronized clocks.
     o Determine the locus of points given by \((\tau_0, \theta_{BS_0})\).
     o Determine the locus of points given by \((\tau_0, \theta_{UE_0})\).
     o Write the equations that determine \(x_{UE}\) and \(\alpha\) from \((\tau_0, \theta_{BS_0}, \theta_{UE_0})\).
   • Assume that there is unknown clock offset \(b\) between BS and UE.
     o Determine the locus of points given by \((\tau_0, \theta_{BS_0}, \theta_{UE_0})\). Is it possible to estimate the UE location? And the UE orientation?

2. Assume that the LOS and one NLOS are received, and also an unknown clock offset \(b\) between BS and UE.
   o Show graphically that it is possible to obtain \(x_{UE}\), \(\alpha\) and \(x_{S_1}\) from \((\tau_0, \theta_{BS_0}, \theta_{UE_0}, \tau_1, \theta_{BS_1}, \theta_{UE_1})\).
   o Therefore, the offset \(b\) can also be determined using only downlink transmission. Hence, show graphically that a wrong choice of \(b\) produces a change in the delays and angles that is inconsistent with the measurements \((\tau_0, \theta_{BS_0}, \theta_{UE_0}, \tau_1, \theta_{BS_1}, \theta_{UE_1})\).
Exercise 2 – Position determination with clock offset - solution

1. Assume only LOS
   • Assume that BS and UE have synchronized clocks.
     o Determine the locus of points given by \((\tau_0, \theta_{BS_0})\).

   a. The value of \(\tau_0\) only LOS determines a circle.
   b. The value of \(\theta_{BS_0}\) determines a line.
   c. The intersection between the circle and the line is the location of the UE.
   d. The UE orientation cannot be determined.
   e. The geometrical problem is the same in the downlink or the uplink.
Exercise 2 – Position determination with clock offset - solution

1. Assume only LOS
   • Assume that BS and UE have synchronized clocks.
     o Determine the locus of points given by \((\tau_0, \theta_{UE_0})\).

   a. The value of \(\tau_0\) only LOS determines a circle.

   b. The value of \(\theta_{UE_0}\) does not add any information in this case. Different locations on the circle result in the same \(\theta_{UE_0}\) for different values of the orientation \(\alpha\), which is unknown.
Exercise 2 – Position determination with clock offset - solution

1. Assume only LOS
   • Assume that BS and UE have synchronized clocks.
     o Write the equations that determine $x_{UE}$ and $\alpha$ from $(\tau_0, \theta_{BS_0}, \theta_{UE_0})$.

   a. The values of $\tau_0$ and $\theta_{BS_0}$ determine the position:

   $$x_{UE} = c \tau_0 \begin{bmatrix} \cos(\theta_{BS_0}) \\ \sin(\theta_{BS_0}) \end{bmatrix}$$

   b. Given the position or the BS-angle, and the UE-angle, one can determine the orientation:

   $$\theta_{UE_0} + \alpha = \theta_{BS_0} + \pi$$
Exercise 2 – Position determination with clock offset - solution

1. Assume only LOS
   • Assume that there is unknown clock offset \( b \) between BS and UE.
     o Determine the locus of points given by \( (\tau_0, \theta_{BS_0}, \theta_{UE_0}) \). Is it possible to estimate the UE location? And the UE orientation?

There is no information about distance. All points in the line lead to the same values of \( \theta_{BS_0} \) and \( \theta_{UE_0} \).
Exercise 2 – Position determination with clock offset - solution

2. Assume that the LOS and one NLOS are received, and also an unknown clock offset \( b \) between BS and UE.

   o Show graphically that it is possible to obtain \( x_{\text{UE}}, \alpha \) and \( x_{S_1} \) from 
   \((\tau_0, \theta_{BS_0}, \theta_{UE_0}, \tau_1, \theta_{BS_1}, \theta_{UE_1})\).

The values of \( \theta_{BS_0} \) and \( \theta_{BS_1} \) determines the lines where the UE and the scatterer are.

The UE orientation can be obtained as:
\[
\alpha = \theta_{BS_0} + \pi - \theta_{UE_0}
\]
2. Assume that the LOS and one NLOS are received, and also an unknown clock offset \( b \) between BS and UE.
   
   o Show graphically that it is possible to obtain \( x_{\text{UE}}, \alpha \) and \( x_{S_1} \) from \( (\tau_0, \theta_{BS_0}, \theta_{UE_0}, \tau_1, \theta_{BS_1}, \theta_{UE_1}) \).

For each point of the line where the UE lies, we can use the difference \( \theta_{UE_0} - \theta_{UE_1} \) to draw segments with that orientation between the two lines.

For each possible location of the UE, the scatterer location can be determined. Therefore, the only independent unknow is now the UE location.
2. Assume that the LOS and one NLOS are received, and also an unknown clock offset $b$ between BS and UE.
   
   - Show graphically that it is possible to obtain $x_{\text{UE}}, \alpha$ and $x_{S_1}$ from $(\tau_0, \theta_{BS_0}, \theta_{UE_0}, \tau_1, \theta_{BS_1}, \theta_{UE_1})$.
   
   Among the possible UE and scatterer locations, we chose the ones for with the path length difference is $\tau_1 - \tau_0$.
   
   Thus, using 6 measurements, we have determined 6 unknowns: $x_{\text{UE}}, \alpha, x_S, b$
Exercise 3 – Position tracking

- We consider a 2D scenario with LOS and two NLOS produced by a scatterer point in unknown position $\mathbf{x}_{S_1}$ and a reflecting wall.

- The reflection point on the wall is denoted as $\mathbf{x}_{S_2}$. 
Exercise 3 – Position tracking

• Note that a virtual anchor (VA) can be associated to each specular reflection. The position is located at the symmetric point to the BS with respect to the wall.

• Write the expression that gives $x_{S_2}$ as a function of $x_{VA_2}$ and $x_{UE}$.

• Assume that:
  o The UE is moving along a straight line with constant velocity.
  o The orientation is constant.
  o The clock offset $b$ is an autoregressive process.

• Write the equations of the dynamic model that connect the parameters $x_{UE}, \alpha$ and $b$ and instant $k$ with the same parameters at instant $k-1$.

• The scatterer point $x_{S_1}$ and the virtual anchor $x_{VA_2}$ are static by definition.

• Write the equations of the dynamic model for $x_{S_1}$ and $x_{VA_2}$.

• Explain which is the advantage of modeling the wall reflection using the VA $x_{VA_2}$ instead of the reflection point $x_{S_2}$.
Exercise 3 – Position tracking

• Write the observation model that links the 9 channel parameters:
  \[(\tau_0, \theta_{BS_0}, \theta_{UE_0}, \tau_1, \theta_{BS_1}, \theta_{UE_1}, \tau_2, \theta_{BS_2}, \theta_{UE_2})\]
  with the location parameters
  \[(x_{UE}, \alpha, b, x_{S_1}, x_{VA_2})\]

• Sketch the formulation of an EKF to track the location parameters using the 9 parameters provided by a channel estimation method.
Exercise 3 – Position tracking - solution

\[ x_{s2} = \beta x_{UE} + (1 - \beta) x_{VA2} \]

where \( \beta \) is obtained by imposing that the distance(UE-VA) is equal to the distance(UE-reflection point) + distance(reflection point-BS).

\[
\beta = \frac{1}{2} \frac{\left| x_{VA2} - x_{UE} \right|^2}{\left( x_{VA2} - x_{BS} \right)^T \left( x_{VA2} - x_{UE} \right)}
\]

where \( \beta \) is obtained by imposing that the distance(UE-VA) is equal to the distance(UE-reflection point) + distance(reflection point-BS).
Exercise 3 – Position tracking - solution

Dynamical model

\[ x_{UE}[k + 1] = x_{UE}[k] + v[k]T + n_{UE} \]
\[ v[k + 1] = v[k] + n_v \]
\[ \alpha[k + 1] = \alpha[k] + n_\alpha \]
\[ b[k + 1] = \gamma b[k] + n_b \]
\[ x_{s_1}[k + 1] = x_{s_1}[k] \]
\[ x_{VA_2}[k + 1] = x_{VA_2}[k] \]