



## POST GRADUATE COURSE

### Information, flows, topology, periodicity, and synchronization in complex networks: Theory and applications

by Murilo da Silva Baptista, University of Aberdeen

4-6 Nov 2013

#### Course schedule and venue:

4 Nov 13:00-15:30 in Lecture Hall TS 101  
5 Nov 10:00-16:00 in Lecture Hall IT 138  
6 Nov 10:00-12:00 in Lecture Hall TS 128

#### ECTS:

TBA

#### Host of the course at DCE:

Pedro Nardelli

### Course description

This course offers an overview of the role of information, flows, topology, and synchronization in complex networks. In the first part of the course the students will learn the theory behind these concepts in general terms. In particular, it will be introduced concepts from Ergodic theory, Dynamical Systems, Complex Networks, Theory of information, Networks Flow Theory, and Synchronization Theory. The second part of the course consists in a wide range of applications such as: topology versus behavior (information, synchronization, and flows) in complex networks, collective almost synchronization, periodic behavior in nonlinear systems, chaos-based wireless communication, chaos-based cryptography (including applications to tablet and smart phone communication), and stability of the power-grid.

### Contents

#### Part I - Theory (6 hours)

**An Introduction to the Theory of Information (30min hour):** I will introduce the main concepts and ideas behind the information Theory and its application for Dynamical systems. In particular, I will introduce Shannon's Entropy, Mutual Information, and Kolmogorov-Sinai entropy.

**An Introduction to Measure Theory and Ergodic Theory (4 hours):** I will introduce the notion of invariant measure, the Ergodic Theorem, and quantities which can be calculated by this theorem, such as Lyapunov exponents, the Kolmogorov-Sinai entropy. I will introduce the Ruelle inequality and the Pesin equality, relating these exponents with the Kolmogorov-Sinai entropy. I will introduce the Perron-Frobenius Theorem, which provides a fundamental calculation of the invariant measure in dynamical systems and that constitutes a fundamental part of Markov models of complex systems.

**An Introduction to the theory of complex networks and network flow (30min hour):** Here the main concepts of theory of graphs and the description of complex networks will be presented. I will also introduce the concept of the network flow theory, which provides ways of calculating how much loads and flows are distributed in a complex network when a given input is given.

**An Introduction to the Theory of Synchronisation (1 hour):** I will introduce a brief historical of the theory of synchronisation of complex networks. I will focus the analysis in case studies: collective synchronisation in Kuramoto networks and complete synchronisation in complex networks of equal nodes (the Master stability function). I will then comment on recent advances of the understanding how synchronisation appears in more general complex networks, such as the ones formed by cluster of subnetworks.

## **Part II – Applications (6 hours)**

**The physics of information transmission, chaos, and predictability in complex networks (1 hour):** The amount of information exchanged per unit of time between two nodes in a dynamical network or between two data sets is a powerful concept for analysing complex systems. This quantity, known as the mutual information rate (MIR) is a fundamental quantity in science, but due to the difficulties in its calculation has so far not attracted too much attention, since when it was first defined by Shannon more than 60 years ago. Its maximal value gives the information capacity between any two sources of information, and remains valid even when the information sources are not stationary, have not statistical stability, and are not memoryless. Therefore, alternative approaches for its calculation or for the calculation of bounds of it are of vital relevance. In this talk, I will show simple formulas to calculate the MIR in dynamical (deterministic) networks or between two time series (not fully deterministic). These formulas link information (the MIR and its bounds) with well known and well defined time-invariant dynamical quantities: the Lyapunov exponents, expansion rates, time recurrences, the speed of correlation decay, and the dimensions, therefore providing a unifying framework to understand the relationship between chaos and predictability in complex systems. As possible applications of these results, I will show how this theoretical approach can be used to reconstruct the networks representing functional and causal relationships between data sets, and discuss conditions of when this functional networks can be used to infer about the real physical network generating the data in a series of complex systems such as the brain, simulated neural networks, coupled oscillators and maps, the DNA, and Hamiltonian systems. I will give special emphasis to shed light into the relationship among sensibility to initial conditions ("chaos") and the predictability in these complex systems.

**Collective Almost Synchronisation in Complex Networks (1 hour):** This lecture introduces the phenomenon of Collective Almost Synchronisation (CAS), which describes a universal way of how patterns can appear in complex networks for small coupling strengths. The CAS phenomenon appears due to the existence of an approximately constant local mean field and is characterised by having nodes with trajectories evolving around periodic stable orbits. Common notion based on statistical knowledge would lead one to interpret the appearance of a local constant mean field as a consequence of the fact that the behaviour of each node is not correlated to the behaviours of the others. Contrary to this common notion, we show that various well known weaker forms of synchronisation (almost, timelag, phase synchronisation, and generalised synchronisation) appear as a result of the onset of an almost constant local mean field. If the memory is formed in a brain by minimising the coupling strength among neurons and maximising the number of possible patterns, then the CAS phenomenon is a plausible explanation for it.

**Periodicity in nonlinear systems (45 min):** The emergence of regular behaviour is one of the most studied topics in nonlinear dynamical systems. It is known that by the changing of an accessible parameter of a chaotic system, chaos can be replaced by a stable periodic behaviour. In this talk, I will review some recent results which accounts to clarifying what are the general conditions under which one can replace chaos into stable periodic behaviour (or vice-versa) by a parameter alteration. I will start by explaining the windows conjecture that describes why for systems that possess  $k$  positive Lyapunov exponents, it is possible to find stable periodic behaviour by altering simultaneously  $k$  control parameters. Then, I will discuss some recent experiments to demonstrate these results in nonlinear electronic circuits, discuss recent works that show that periodic windows can be used to understand the behaviour of a different class of systems, varying from mechanical systems to bird population dynamics, and discuss other works such as in Refs. that clarify the relationship between the structure of periodic windows and homoclinic bifurcations.

**Wireless communication with chaos (45 min):** The modern world fully relies on wireless communication. Because of intrinsic physical constraints of the wireless physical media (multipath, damping, and filtering), signals carrying information are strongly modified, preventing information from being transmitted with a high bit rate. We show that, though a chaotic signal is strongly modified by the wireless physical media, its Lyapunov exponents remain unaltered, suggesting that the information transmitted is not modified by the channel. For some particular chaotic signals, we have indeed proved that the dynamic description of both the transmitted and the received signals is identical and shown that the capacity of the chaos-based wireless channel is unaffected by the multipath propagation of the physical media. These physical properties of chaotic signals warrant an effective chaos-based wireless communication system.

**Cryptography with chaos (1 hour):** In this lecture I will present the fundamentals of a cryptographic method based on a hyperchaotic system and a protocol which inherits some properties of the quantum cryptography that can be straightforwardly applied on the existing communication systems of nonoptical communication channels. It is an appropriate tool to provide security on software applications for VoIP, as in Skype, dedicated to voice communication through Internet. This would enable that an information packet be sent through Internet preventing attacks with strategies similar to that employed if this same packet is transferred in an optical channel under a quantum cryptographic scheme. This method relies on fundamental properties possessed by chaotic signals and coupled chaotic systems. Some of these properties have never been explored in secure communications. This talk will introduce also how this idea can be applied to Applets and Smartphones. Finally, I will present how coupled networks can be used to transmit information in an encrypted way mimicking the way the brain process information.

**The stability of the power grid (45 min):** Synchronization between spatially distributed nodes of a power-grid is a crucial requirement for its proper operation. Using a Kuramoto-like network as a realistic physical model for the distribution of electrical power in a power-grid, we obtain coupling strengths and topological characteristics that favor the synchronous state of those technological networks. Power-grids are highly heterogeneous. They are composed of different classes of nodes and each node behaves differently. We show in this work that if a power-grid is extensive and nodes are highly connected, the coupling strength that leads to synchronization depends mainly on the eigenvalues of the Laplacian matrix, as it happens in homogeneous networks composed of equal nodes. On the other hand, if a power-grid is sparsely connected, the coupling strength that leads to synchronization is also strongly related to the correlation coefficient of the network, which means that a high number of connections between similar nodes (two power plants or two consumer centers) disfavor the synchronizability of the power-grid.

**Structure and function in flow networks (45 min):** In this lecture, I will present a unified approach for the fundamental relationship between structure and function in flow networks by solving analytically the voltages in a resistor network, transforming the network structure to an effective all-to-all topology, and then measuring the resultant flows. Moreover, it defines a way to study the structural resilience of the graph and to detect possible communities.

## **Evaluation**

The students can choose between solving a list of exercises based on the theoretical part of the course (part I) or write an essay (topic to be agreed during the course) related to topics introduced in second part of the course.

## **Curriculum vitae**

About the lecturer: Dr. Murilo da Silva Baptista has a bachelor degree in physics from University of São Paulo (1992) and a PhD degree in physics from the same institution (1996). He also held a postdoctoral position at University System of Maryland (1997-1999), at University of São Paulo (1999-2004) and at University of Potsdam (2004-2006). He was also a visiting researcher at National Institute of Applied Optics - Italy (2002-2003), Max-Planck Institute for the Physics of Complex Systems (2006-2008), and was a guest assistant lecturer of the Center for Mathematics, University of Porto (2008-2009). He joined the University of Aberdeen in 2009, where he is currently Senior Lecturer at the Institute for Complex Systems and Mathematical Biology and at the recently created Institute of Pure and Applied Mathematics. He has experience in the large multidisciplinary area of modern physics and applied mathematics, which includes the areas of Complex Networks, Information, Dynamical Systems, Synchronisation, Chaos, Chaos control, time-series analysis, and modeling of Biological systems.