Analyyttinen mekaniikka 763310A
Apukaavoja

$$L = T - V$$

$$S = \int_{t_1}^{t_2} L\, dt \tag{1}$$

Cylindrical coordinates (ρ, ϕ, z)

$$T = \frac{1}{2} m \left(\dot{\rho}^2 + \rho^2 \dot{\phi}^2 + \dot{z}^2 \right). \tag{2}$$

Spherical coordinates (r, θ, ϕ)

$$T = \frac{1}{2} m \left(\dot{r}^2 + r^2 \dot{\theta}^2 + r^2 \sin^2\theta \dot{\phi}^2 \right). \tag{3}$$

Electromagnetic potential

$$V(\boldsymbol{r}, \boldsymbol{v}, t) = q \left[\varphi(\boldsymbol{r}, t) - \boldsymbol{v} \cdot \boldsymbol{A}(\boldsymbol{r}, t) \right]. \tag{4}$$

Lagrange equations with differential constraints

$$\sum_{i=1}^{n} a_{li} dq_i + a_{lt} dt = 0, \ (l = 1, \ldots, m), \tag{5}$$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = \sum_{l=1}^{m} \lambda_l a_{li}, \quad (i = 1, \ldots, n), \tag{6}$$

$$\sum_{i=1}^{n} a_{li} \dot{q}_i + a_{lt} = 0, \ (l = 1, \ldots, m). \tag{7}$$

Differential scattering cross section

$$\frac{d\sigma}{d\Omega}(\theta) = \frac{b}{\sin\theta} \left| \frac{db}{d\theta} \right|. \tag{8}$$

Small oscillations

$$L = \frac{1}{2} \sum_{ij} (A_{ij} \dot{\eta}_i \dot{\eta}_j - v_{ij} \eta_i \eta_j), \tag{9}$$

$$\det(\mathsf{v} - \omega^2 \mathsf{A}) = 0. \tag{10}$$

Non-inertial frame

$$\left(\frac{d}{dt} \right)_{\text{inertial}} = \left(\frac{d}{dt} \right)_{\text{kappale}} + \boldsymbol{\omega} \times, \tag{11}$$

$$m \left(\frac{d^2 \boldsymbol{r}}{dt^2} \right)_{\text{kappale}} = \boldsymbol{F}^{\text{e}} - m \left(\frac{d^2 \boldsymbol{R}}{dt^2} \right)_{\text{inertial}}$$
$$-2m\boldsymbol{\omega} \times \left(\frac{d\boldsymbol{r}}{dt} \right)_{\text{kappale}} - m \frac{d\boldsymbol{\omega}}{dt} \times \boldsymbol{r} - m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}).$$

Moment of inertia

$$I_{ij} = \int d^3 r \rho \left(\delta_{ij} r^2 - r_i r_j \right), \tag{12}$$

$$T = \frac{1}{2} \sum_{i=1}^{3} \sum_{j=1}^{3} \omega_i I_{ij} \omega_j, \tag{13}$$

$$L_i = \sum_{j=1}^{3} I_{ij} \omega_j. \tag{14}$$

Euler angles α, β and γ:

$$\begin{aligned} \omega_1 &= -\dot{\alpha} \sin\beta \cos\gamma + \dot{\beta} \sin\gamma, \\ \omega_2 &= \dot{\alpha} \sin\beta \sin\gamma + \dot{\beta} \cos\gamma, \\ \omega_3 &= \dot{\alpha} \cos\beta + \dot{\gamma}. \end{aligned} \tag{15}$$

Poisson brackets

$$[A, B]_{\text{PB}} \equiv \sum_i \left(\frac{\partial A}{\partial q_i} \frac{\partial B}{\partial p_i} - \frac{\partial A}{\partial p_i} \frac{\partial B}{\partial q_i} \right), \tag{16}$$

$$\frac{dA}{dt} = -[H, A]_{\text{PB}} + \frac{\partial A}{\partial t}, \tag{17}$$

$$[A, B]_{\text{PB}} \to -\frac{i}{\hbar} [\check{A}, \check{B}]. \tag{18}$$

Equation of continuity

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v}) = 0. \tag{19}$$