56. Laske esimerkin jouseen kiinnitetyn kappaleen hetkellinen nopeus.

```
Clear[m, x, k, a]
DSolve[{m x''[t] == -k x[t], x[0] == a, x'[0] == 0}, x[t], t]
```

$\left\{\left\{x[t] \to a\,\mathrm{Cos}\left[\frac{\sqrt{k}\ t}{\sqrt{m}}\right]\right\}\right\}$

```
x[t_] = x[t] /. %[[1]]
```

$a\,\mathrm{Cos}\left[\frac{\sqrt{k}\ t}{\sqrt{m}}\right]$

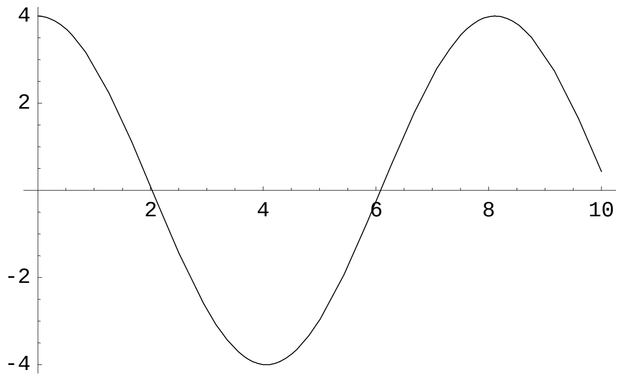
```
v = x'[t]
```

$-\frac{a\sqrt{k}\ \mathrm{Sin}\left[\frac{\sqrt{k}\ t}{\sqrt{m}}\right]}{\sqrt{m}}$

57. Kirjoita tiedostoon Mathematica-ohjelma, joka antaa a:lle, k:lle ja m:lle järkevät arvot,ratkaisee yhtälön kuten edellä ja piirtää kuvaajan kappaleen poikkeamasta ajan funktiona.

```
Clear[m, x, k, a]
a = 4;
k = 1.2;
m = 2;
ratk = DSolve[{m x''[t] == -k x[t], x[0] == a, x'[0] == 0}, x[t], t]
Plot[x[t] /. ratk, {t, 0, 10}]
```

{{x[t] → 4. Cos[0.774597 t] + 0. Sin[0.774597 t]}}



- Graphics -

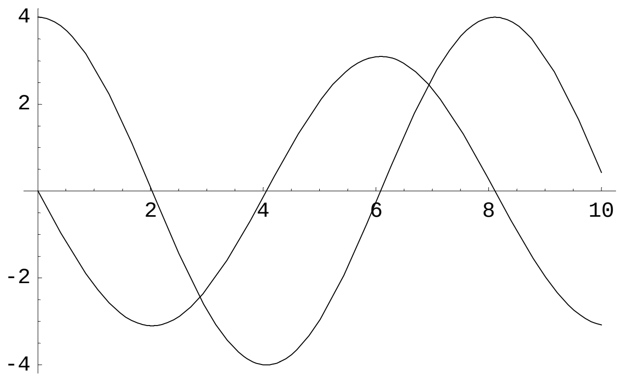
58. Lisää ohjelmaan pätkä, joka laskee kappaleen hetkellisen nopeuden ja piirtää sen samaan kuvaan poikkeaman kanssa.Tulkitse kuvaa. (Huomaa, ettet voi viitata tuloksiin %n-menetelmällä ohjelman sisällä!)

```
Clear[m, x, k, a]
a = 4;
k = 1.2;
m = 2;
ratk = DSolve[{m x''[t] == -k x[t], x[0] == a, x'[0] == 0}, x[t], t]
x[t_] = x[t] /. ratk[[1]]
v = x'[t]
Plot[{x[t], v}, {t, 0, 10}]
```

```
{{x[t] → 4. Cos[0.774597 t] + 0. Sin[0.774597 t]}}
```

```
4. Cos[0.774597 t] + 0. Sin[0.774597 t]
```

```
0. Cos[0.774597 t] - 3.09839 Sin[0.774597 t]
```




```
- Graphics -
```

59. Ratkaise jousen paikka ajan funktiona.

```
Clear[m, x, k, a, g]
ratk = DSolve[{m x''[t] == -k x[t] - g x'[t], x[0] == a, x'[0] == 0}, x[t], t]
```

$$\left\{\left\{x[t] \to \frac{1}{2\sqrt{g^2-4\,k\,m}}\left(a\left(-e^{\frac{\left(-g-\sqrt{g^2-4\,k\,m}\right)t}{2\,m}}\,g+e^{\frac{\left(-g+\sqrt{g^2-4\,k\,m}\right)t}{2\,m}}\,g+e^{\frac{\left(-g-\sqrt{g^2-4\,k\,m}\right)t}{2\,m}}\sqrt{g^2-4\,k\,m}+e^{\frac{\left(-g+\sqrt{g^2-4\,k\,m}\right)t}{2\,m}}\sqrt{g^2-4\,k\,m}\right)\right)\right\}\right\}$$

```
x[t_] = x[t] /. ratk[[1]]
```

$$\frac{1}{2\sqrt{g^2-4\,k\,m}}\left(a\left(-e^{\frac{\left(-g-\sqrt{g^2-4\,k\,m}\right)t}{2\,m}}\,g+e^{\frac{\left(-g+\sqrt{g^2-4\,k\,m}\right)t}{2\,m}}\,g+e^{\frac{\left(-g-\sqrt{g^2-4\,k\,m}\right)t}{2\,m}}\sqrt{g^2-4\,k\,m}+e^{\frac{\left(-g+\sqrt{g^2-4\,k\,m}\right)t}{2\,m}}\sqrt{g^2-4\,k\,m}\right)\right)$$

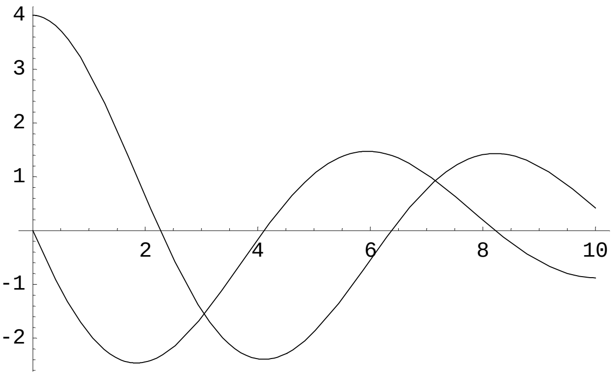
60. Kuten kitkattomassa tapauksessa, piirrä paikan ja nopeuden aikakehitys samaan kuvaajaan. Mitä eroja havaitset?

```
v = x'[t]
```

$$\frac{1}{2\sqrt{g^2-4km}}\left(a\left(-\frac{e^{\frac{\left(-g-\sqrt{g^2-4km}\right)t}{2m}}g\left(-g-\sqrt{g^2-4km}\right)}{2m}+\frac{e^{\frac{\left(-g-\sqrt{g^2-4km}\right)t}{2m}}\sqrt{g^2-4km}\left(-g-\sqrt{g^2-4km}\right)}{2m}+\frac{e^{\frac{\left(-g+\sqrt{g^2-4km}\right)t}{2m}}g\left(-g+\sqrt{g^2-4km}\right)}{2m}+\frac{e^{\frac{\left(-g+\sqrt{g^2-4km}\right)t}{2m}}\sqrt{g^2-4km}\left(-g+\sqrt{g^2-4km}\right)}{2m}\right)\right)$$

```
a = 4;
k = 1.2;
m = 2;
g = 0.5;
```

```
Plot[{x[t], v}, {t, 0, 10}]
```



- Graphics -

Luonnollisesti, kun vaimennustekijä lisätään, niin värähtely vaimenee!

61. Eliminoi vakiot asettamalla varaukselle q ja virralle q' sopivat alkuehdot, ja tutki saamaasi lopputulosta kuvin.Vaihtele komponenttien arvoja ja tutki systeemin käyttäytymistä.

```
Clear[c, l, q]
ratk = DSolve[{q[t] / c + l q''[t] == 0, q[0] == 10, q'[0] == 0}, q[t], t]
```

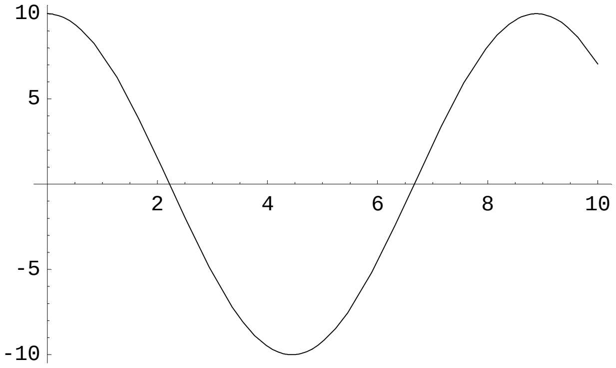
$$\left\{\left\{q[t]\to 10\,\mathrm{Cos}\left[\frac{t}{\sqrt{c}\,\sqrt{l}}\right]\right\}\right\}$$

```
q[t_] = q[t] /. ratk[[1]]
```

$$10\,\mathrm{Cos}\left[\frac{t}{\sqrt{c}\,\sqrt{l}}\right]$$

```
c = 1; l = 2;
```

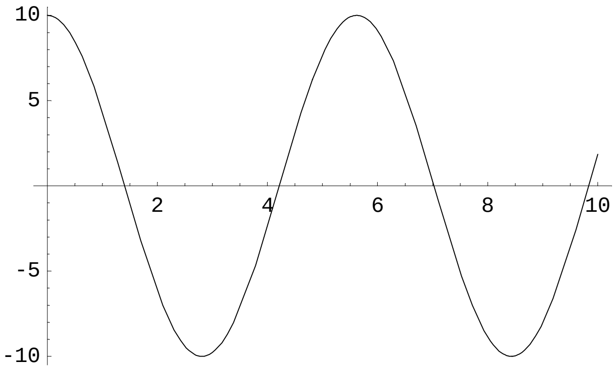
Plot[**q**[**t**], {**t**, **0**, **10**}]



- Graphics -

c = 0.2; l = 4;

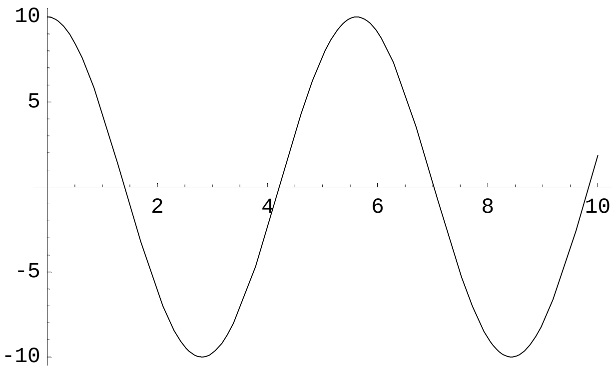
Plot[**q**[**t**], {**t**, **0**, **10**}]



- Graphics -

c = 4; l = 0.2;

Plot[**q**[**t**], {**t**, **0**, **10**}]



- Graphics -

62. Eliminoi vakiot kuten LC-piirien tapauksessa, ja tutki saamaasi lopputulosta kuvin. Vaihtele komponenttien arvoja ja tutki systeemin käyttäytymistä.

```
Clear[q, l, r, c]
ratk = DSolve[{l q''[t] + r q'[t] + q[t] / c == 0, q[0] == 10, q'[0] == 0}, q[t], t]
```

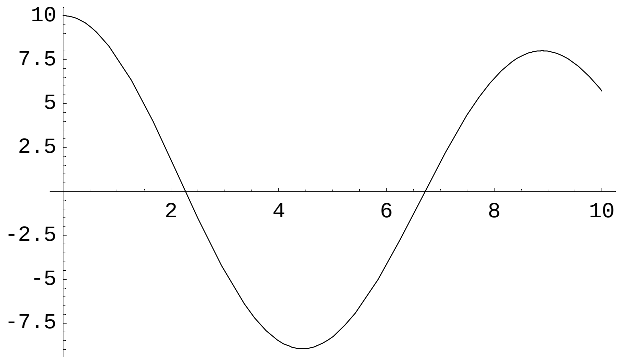
$$\left\{\left\{q[t] \to \frac{1}{\sqrt{-4\,l + c\,r^2}}\left(5\left(-\sqrt{c}\,e^{\frac{\left(-\sqrt{c}\,r-\sqrt{-4\,l+c\,r^2}\right)t}{2\sqrt{c}\,l}}\,r + \sqrt{c}\,e^{\frac{\left(-\sqrt{c}\,r+\sqrt{-4\,l+c\,r^2}\right)t}{2\sqrt{c}\,l}}\,r + e^{\frac{\left(-\sqrt{c}\,r-\sqrt{-4\,l+c\,r^2}\right)t}{2\sqrt{c}\,l}}\sqrt{-4\,l+c\,r^2} + e^{\frac{\left(-\sqrt{c}\,r+\sqrt{-4\,l+c\,r^2}\right)t}{2\sqrt{c}\,l}}\sqrt{-4\,l+c\,r^2}\right)\right)\right\}\right\}$$

```
q[t_] = q[t] /. ratk[[1]]
```

$$\frac{1}{\sqrt{-4\,l + c\,r^2}}\left(5\left(-\sqrt{c}\,e^{\frac{\left(-\sqrt{c}\,r-\sqrt{-4\,l+c\,r^2}\right)t}{2\sqrt{c}\,l}}\,r + \sqrt{c}\,e^{\frac{\left(-\sqrt{c}\,r+\sqrt{-4\,l+c\,r^2}\right)t}{2\sqrt{c}\,l}}\,r + e^{\frac{\left(-\sqrt{c}\,r-\sqrt{-4\,l+c\,r^2}\right)t}{2\sqrt{c}\,l}}\sqrt{-4\,l+c\,r^2} + e^{\frac{\left(-\sqrt{c}\,r+\sqrt{-4\,l+c\,r^2}\right)t}{2\sqrt{c}\,l}}\sqrt{-4\,l+c\,r^2}\right)\right)$$

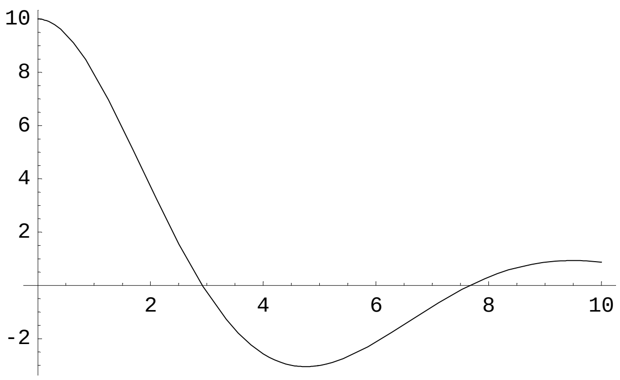
```
c = 1; l = 2; r = 0.1;
```

```
Plot[q[t], {t, 0, 10}]
```



- Graphics -

```
c = 1; l = 2; r = 1;
Plot[q[t], {t, 0, 10}]
```



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