

18. Ratkaise yleinen kolmannen asteen yhtälö $ax^3+bx^2+cx+d=0$ käyttämällä reduce-komentoa.

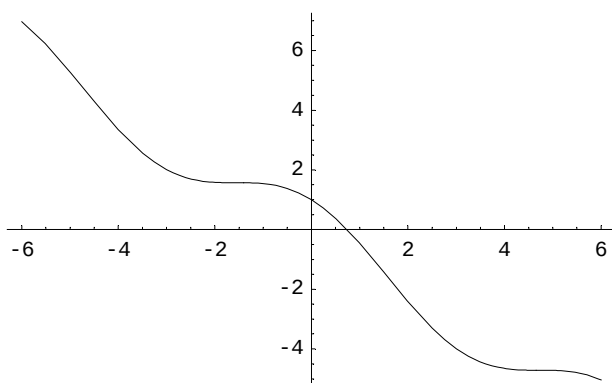
Clear[a, b, c, d, x]

Reduce[a x³ + b x² + c x + d == 0, x]

$$\begin{aligned}
 x = & -\frac{b}{3a} - \left(2^{1/3} (-b^2 + 3ac)\right) \Bigg/ \left(3a \left(-2b^3 + 9abc - 27a^2d + \sqrt{4(-b^2 + 3ac)^3 + (-2b^3 + 9abc - 27a^2d)^2}\right)^{1/3}\right) + \\
 & \frac{1}{3 \cdot 2^{1/3} a} \left(\left(-2b^3 + 9abc - 27a^2d + \sqrt{4(-b^2 + 3ac)^3 + (-2b^3 + 9abc - 27a^2d)^2}\right)^{1/3}\right) \& \& \\
 a \neq 0 \mid \mid x = & -\frac{b}{3a} + \left((1 + i\sqrt{3}) (-b^2 + 3ac)\right) \Bigg/ \left(3 \cdot 2^{2/3} a \right. \\
 & \left. \left(-2b^3 + 9abc - 27a^2d + \sqrt{4(-b^2 + 3ac)^3 + (-2b^3 + 9abc - 27a^2d)^2}\right)^{1/3}\right) - \frac{1}{6 \cdot 2^{1/3} a} \\
 & \left((1 - i\sqrt{3}) (-2b^3 + 9abc - 27a^2d + \sqrt{4(-b^2 + 3ac)^3 + (-2b^3 + 9abc - 27a^2d)^2})^{1/3}\right) \& \& \\
 a \neq 0 \mid \mid x = & -\frac{b}{3a} + \left((1 - i\sqrt{3}) (-b^2 + 3ac)\right) \Bigg/ \left(3 \cdot 2^{2/3} a \right. \\
 & \left. \left(-2b^3 + 9abc - 27a^2d + \sqrt{4(-b^2 + 3ac)^3 + (-2b^3 + 9abc - 27a^2d)^2}\right)^{1/3}\right) - \frac{1}{6 \cdot 2^{1/3} a} \\
 & \left((1 + i\sqrt{3}) (-2b^3 + 9abc - 27a^2d + \sqrt{4(-b^2 + 3ac)^3 + (-2b^3 + 9abc - 27a^2d)^2})^{1/3}\right) \& \& \\
 a \neq 0 \mid \mid a = 0 \& \& x = & \frac{-c - \sqrt{c^2 - 4bd}}{2b} \& \& b \neq 0 \mid \mid \\
 a = 0 \& \& x = & \frac{-c + \sqrt{c^2 - 4bd}}{2b} \& \& b \neq 0 \mid \mid \\
 a = 0 \& \& b = 0 \& \& \\
 c = 0 \& \& d = 0 \mid \mid \\
 a = 0 \& \& b = 0 \& \& x = & -\frac{d}{c} \& \& c \neq 0
 \end{aligned}$$

19. Etsi yhtälön $\cos(x)=x$ ratkaisu.

Plot[Cos[x] - x, {x, -6, 6}]



- Graphics -

FindRoot[Cos[x] == x, {x, -2, 1}, AccuracyGoal → 200, MaxIterations → 90]

{x → 0.739085}

Cos[x] - x /. %

0.

20. Ratkaise seuraava yhtälöryhmä

$$3x+y=2$$

$$y+2z=2$$

$$x+z=1$$

symbolisesti ja numeerisesti.

Solve[{3 x + y == 2, y + 2 z == 2, x + z == 1}, {x, y, z}]

NSolve[{3 x + y == 2, y + 2 z == 2, x + z == 1}, {x, y, z}]

$\left\{ \left\{ x \rightarrow \frac{2}{5}, y \rightarrow \frac{4}{5}, z \rightarrow \frac{3}{5} \right\} \right\}$

$\{ \{ x \rightarrow 0.4, y \rightarrow 0.8, z \rightarrow 0.6 \} \}$

21. Etsi yhtälöille

$$x^2+3x-1=0$$

$$3x^2+5x+1=0$$

symboliset ja numeeriset ratkaisut.

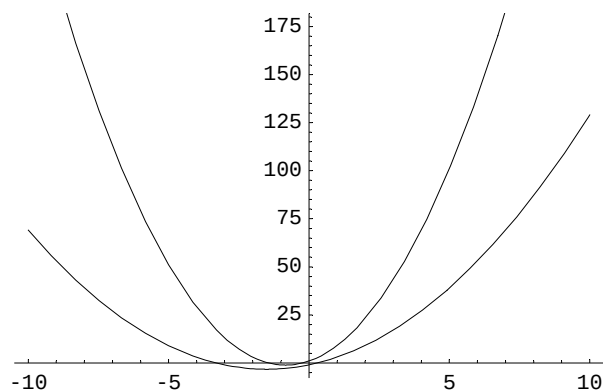
Solve[{x^2 + 3 x - 1 == 0, 3 x^2 + 5 x + 1 == 0}, x]

NSolve[{x^2 + 3 x - 1 == 0, 3 x^2 + 5 x + 1 == 0}, x]

{}

NSolve[{-1 + 3 x + x^2 == 0, 1 + 5 x + 3 x^2 == 0}, x]

Plot[{x^2 + 3 x - 1, 3 x^2 + 5 x + 1}, {x, -10, 10}]



- Graphics -

Kuten kuvasta näkyy, yhtälöparilla ei ole ratkaisuja.

22. Ratkaise yhtälöryhmä

$$x^2+5x-4y=0$$

$$2x-y=3$$

Solve[{x^2 + 5 x - 4 y == 0, 2 x - y == 3}, {x, y}]

$\left\{ \left\{ y \rightarrow -i\sqrt{39}, x \rightarrow \frac{1}{2} (3 - i\sqrt{39}) \right\}, \left\{ y \rightarrow i\sqrt{39}, x \rightarrow \frac{1}{2} (3 + i\sqrt{39}) \right\} \right\}$

```
NSolve[{x^2 + 5 x - 4 y == 0, 2 x - y == 3}, {x, y}]
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{ {y → 0. + 6.245 i, x → 1.5 + 3.1225 i}, {y → 0. - 6.245 i, x → 1.5 - 3.1225 i}}
```

23. Ratkaise lineaarinen yhtälöryhmä

$$3x_1 + x_2 + 4x_3 = 2$$

$$5x_1 + x_2 + 2x_3 = 3$$

$$x_1 - 3x_2 - 2x_3 = 1$$

```
m = {{3, 1, 4}, {5, 1, 2}, {1, -3, -2}}
```

```
b = {2, 3, 1}
```

```
LinearSolve[m, b]
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```
{ {3, 1, 4}, {5, 1, 2}, {1, -3, -2}}
```

```
{2, 3, 1}
```

```
{  $\frac{3}{5}$ ,  $-\frac{1}{5}$ ,  $\frac{1}{10}$  }
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