The king of snooker

Little Niko-Petteri wants to become the new king of snooker. Insecure of his skills, he uses physics to calculate how he must make his shots. It is your task in this excersise to do the necessary math in Mathematica.

The table is setup as in Fig. 1. Niko-Petteri must make his shot so that he gets the black ball into the corner pocket. The balls are alike and their mass is $M = 150$ g and radius 2.7 cm.

Task 1.

Your first task is to calculate the initial velocity that the black ball needs inorder to roll into the pocket. We assume that the force slowing the ball down is constant and given by the rolling friction between the ball and the cloth,

$$F_r = \mu_r M g, \tag{1}$$

where the coefficient of rolling friction $\mu_r = 0.01$ and $g = 9.81$ m/s^2 is the acceleration due to Earth's gravity. As it rolls on the cloth, the ball does the work W,

$$W = F_r s, \tag{2}$$

where s is the distance traveled by the ball (See Fig. 1). The kinetic energy of the ball is the sum of its translational and rotational energies, T^{trans} and T^{rot}, respectively. The total kinetic energy is

$$T = T^{\text{trans}} + T^{\text{rot}} = \frac{1}{2}Mv^2 + \frac{1}{2}J\omega^2, \tag{3}$$

where v is the velocity and ω the angular velocity. The quantity J is the moment of inertia of the ball,

$$J = \frac{2}{5}MR^2. \tag{4}$$

If the ball rolls without slipping, its angular velocity is

$$\omega = \frac{v}{R}. \tag{5}$$

So you task is:

1. Solve the equation $W = T$ with respect to velocity v both symbolically and numerically.
2. Plot the total kinetic energy of the ball as a function of the velocity.

Task 2.

Your second task is to solve the velocities of the white and black ball, so that the black ball rolls to the pocket. We assume, that the kinetic energy acquired is enough for the ball to reach the bag.

Let the momentum and the kinetic energy of the white ball just before the impact be $\boldsymbol{p}_w = m\boldsymbol{v}_w$ and $T_w = 1/2mv_w^2$, and after the impact $\boldsymbol{p}'_w = m\boldsymbol{v}'_w$ and $T_w{}' = 1/2mv'^{2}_w$. The black ball at rest in the begininng, so that $\boldsymbol{p}'_b = 0$ and $T_b = 0$. As for the white ball, we denote the momentum and energy of the ball by $\boldsymbol{p}'_b = m\boldsymbol{v}'_b$ and $T_b{}' = 1/2mv'^{2}_b$. Assume that the collision is not fully elastic, but that a small fraction of the kinetic energy, $E_{\text{inel}} > 0$, is lost after the collision. When there is no friction between the balls, the rotational energies of the balls is conserved, and thus vanishes from the equations.

$$\boldsymbol{p}_w = \boldsymbol{p}'_w + \boldsymbol{p}'_b, \tag{6}$$

$$T_w^{\text{trans}} = T_w^{\text{trans}\prime} + T_b^{\text{trans}\prime} - E_{\text{inel}}. \tag{7}$$

So that the ball would go directly into the corner pocket, the velocity vector $\boldsymbol{v}_b{}'$ must make a 225^o degree angle with the x–axis. This condition is satisfied when $v'_{bx} = v'_{by} < 0$, as depicted in Fig. 1. Together with Eqs. (7) we obtain the set of equations

$$
\begin{aligned}
mv_{wx} &= mv'_{wx} + mv'_{bx} && \text{(8a)}\\
mv_{wy} &= mv'_{wy} + mv'_{by} && \text{(8b)}\\
\frac{1}{2}m(v_{wx}^2 + v_{wy}^2) &= \frac{1}{2}m({v'_{wx}}^2 + {v'_{wy}}^2) + \frac{1}{2}m({v'_{bx}}^2 + {v'_{by}}^2) - E_{\text{inel}} && \text{(8c)}\\
v'_{bx} &= v'_{by}. && \text{(8d)}
\end{aligned}
$$

In addition to the velocities, we are interested in the angle that the velocities of the white and black ball make after the collision. This can be solved from the scalar product

$$\boldsymbol{v}_b{}' \cdot \boldsymbol{v}_w{}' = |\boldsymbol{v}_b||\boldsymbol{v}_w| \cos\theta,$$

where θ is the angle that we are interested in. When expanded, the above equation reads

$$\frac{v'_{bx}v'_{wx} + v'_{by}v'_{wy}}{\sqrt{{v'_{bx}}^2 + {v'_{by}}^2}\sqrt{{v'_{wx}}^2 + {v'_{wy}}^2}} = \cos\theta. \qquad (9)$$

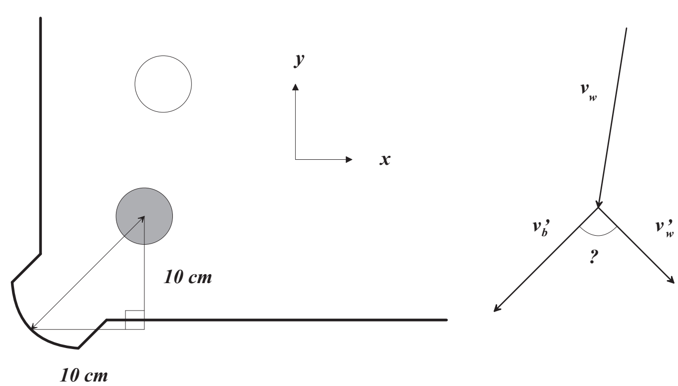
Do the following:

1. For starters, let us assume that the collision is perfectly elastic, that is, $E_{\text{inel}} = 0$. Analytically solve the end velocities of the white and black ball from the equations (8). Substitute your solution into the equation (9) and from that, solve the angle θ. As your answer, give (a) the end velocities of the balls as functions of the initial velocity of the white ball and (b) the angle θ between the end velocities.

2. Let us now assume that the white ball hits the black ball at a velocity $\boldsymbol{v}_w = (-0.2, -1.25)$ m/s, and that one percent of the kinetic energy is lost in the collision

$$E_{\text{inel}} = 0.01 \cdot T_w^{\text{trans}}.$$

   Solve the set of equations (8) numerically. Substitute the solution you obtained into the equation (9) and numerically solve the angle θ. As your answer give (a) the end velocities of the balls for the given $\boldsymbol{v}_w$ and (b) the angle between the end velocities of the balls. By how many degrees does the result of part (1) change?



Kuva 1: The situation Niko-Petteri is facing and the ball velocities, schematically.