Practice Work 1 – Collisions

This practice work is divided into two sections. First, we deal with 1D-collisions where the colliding particles move along a straight line, before and after the collision. That part shouldn't cause too much trouble. The second part is about two-dimensional collisions. To make it easier, I will present a short summary of the theory related to it.

Consider two particles, labelled 1 and 2, with masses m_1 and m_2 and velocities (before the collision) $\mathbf{v}_1$ and $\mathbf{v}_2$. Assume that they collide. They bounce off each other and move away with velocities $\mathbf{v}'_1$ and $\mathbf{v}'_2$. First of all, the sum of their momenta $\mathbf{p} = m\mathbf{v}$ is conserved in the collision,

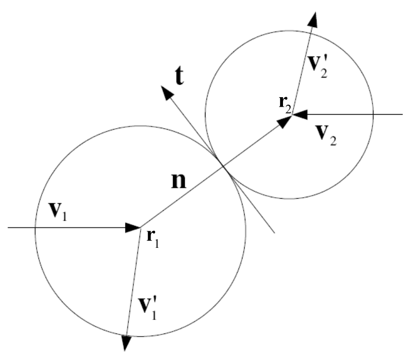
$$m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = m_1\mathbf{v}'_1 + m_2\mathbf{v}'_2 \tag{1}$$

Furthermore, if the collision is elastic, the total kinetic energy of the particles is conserved:

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2 \tag{2}$$

where v_i is the speed of particle i, i.e. the length of the velocity vector $\mathbf{v}_i$.

When the collision is not head-on, the particles will scatter off each other to some direction which depends on the way they collided, and also on their shape. In this practice work, we investigate the collisions of hard, perfectly round spheres, in which case the movement of the particles is restricted to a plane, i.e. to two dimensions. This plane is called the *plane of collision*.



If the collision between the spheres is "perfect" in the sense that they don't rub against each other, a plane can be pictured separating the trajectories of the spheres. This plane is perpendicular to the plane of collision and the intersection of these planes is a line, directed along the vector $\mathbf{t}$.

The trick in investigating a two-dimensional collision problem is to divide the initial velocities into *normal* and *tangential* components, i.e. components along the vectors $\mathbf{n}$ and $\mathbf{t}$. The tangential components will remain constant during the collision, and the normal components are transformed according to the one-dimensional formulas, which were discussed above.

The components can be calculated as scalar products

$$v_{i,n} = \mathbf{v}_i \cdot \hat{\mathbf{n}} \tag{3}$$

and

$$v_{i,t} = \mathbf{v}_i \cdot \hat{\mathbf{t}} \tag{4}$$

where $\hat{\mathbf{n}} = \mathbf{n}/n$ and $\hat{\mathbf{t}} = \mathbf{t}/t$ are unit vectors. The normal vector $\mathbf{n}$ can be constructed from the difference of the position vectors of the centers of the spheres at the time of collision, $\mathbf{n} = \mathbf{r}_2 - \mathbf{r}_1$ and the tangential vector can be calculated from the normal vector by rotating it ninety degrees in the plane of collision. This is achieved by

$$\mathbf{t} = R\mathbf{n} \tag{5}$$

where

$$R = \begin{pmatrix} \cos\pi/2 & -\sin\pi/2 \\ \sin\pi/2 & \cos\pi/2 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{6}$$

is the rotation matrix.

When the tangential and normal components of a vector are known, the "original" x and y components can be obtained by calculating the projection of the velocity vector onto the coordinate axes. To do this, the unit vectors $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ must be presented in terms of $\hat{\mathbf{n}}$ and $\hat{\mathbf{t}}$. You should be able to figure out how to do this by referring to the above.

There are other ways to do the above coordinate transformations. If you choose to use other mathematical methods than those given here, you must give a detailed derivation the formulas in your practice work report.

1. Consider a one-dimensional (head-on) collision along the x-axis. From equations (1) and (2), solve the velocities $\mathbf{v}'_1$ and $\mathbf{v}'_2$. In this case you don't have to worry about the vector nature of the velocities, since they only have one component, before and after the collision.

   Then test your collision simulator and your *Mathematica* skills in the following excercises:

   a) Particle A (mass 150g), travelling with the speed 10,2m/s, collides head-on with particle B (mass 2,23kg), initially at rest. Assuming that the collision can be regarded as elastic, find the speeds of both of the particles after the collision.

   b) Consider the same particles as in the previous excercise. What should be the initial speed of particle A so that its final speed (i.e. after the collision) would be 4,5m/s, away from particle B. Assume again that particle B is initially at rest.

   c) Again, the same particles. This time, particle B moves initially away from particle A (i.e. in the same direction) with speed v_2. Particle A moves initially with the same speed as in a). What should the speed v_2 be, so that particle A stays still after the collision?

2. Now it's time to attack the two-dimensional problem. Let the radii of the colliding spheres be R_1 and R_2. You can assume that the initial velocities are parallel (or antiparallel). As a parameter describing the relative orientation of the spheres you can use the perpendicular distance of the lines determined by $\mathbf{v}_1$ and $\mathbf{v}_2$, call it d. Note that d must be allowed to have positive and negative values. Right away you can deduce that no collision will happen if $|d| \geq R_1 + R_2$, so this condition determines the parameter range for d. The normal vector is

$$\mathbf{n} = \hat{\mathbf{x}}\sqrt{(R_1 + R_2)^2 - d^2} + \hat{\mathbf{y}}d \tag{7}$$

and the tangential vector can be calculated using (5). Then calculate the normal and tangential components of the initial velocities using (3) and (4). The tangential components remain unaltered in the collision and the normal components obey the formula derived in excercise 1.

a) Consider two particles with masses 1,50kg and 1,25kg and radii 15cm and 13cm, respectively. They collide at speeds $v_1 = 1.5$m/s and $v_2 = 1.25$m/s, with $d = -10cm$. Calculate their velocities after the collision.

b) Consider two particles with masses 0,01kg and 10.0kg and radii 0,1cm and 20,0cm, respectively. The large particle is initially at rest and the small particle collides with it at speed 1,00m/s. Define a function, which gives the angle between the velocity of the small particle after the collision and before the collision and plot the angle as a function of the parameter d. *Hint:* calculate the scalar product of the vectors to get the angle.

c) Consider two identical particles both moving initially at 1,00m/s and carry out the same task as in b). Then also plot the angle between the final velocities of the particles, again as a function of d.

Carry out the above excercises. Do *all* of your calculations using *Mathematica*. Make a practice work report in which you present

- The assignment in your own words (not this pdf-file!)
- Your *thoroughly commented* calculations
- The final results also in a separate section

Return the report in printed form to the *in*-box in the theoretical physics lobby. On the first page of your report, write *only* the following:

- Course name and practice work number
- Your name and e-mail address
- Date of return

If you need help, the practice work sessions are on **tuesdays** at **8–11** in the classroom KO130.