Practice Work 2
Data Analysis

In this practice work you get to do some interpolation, data fitting and error estimation. The experimental setup described might be a touch unrealistic, but don't stress about that.

The Setup

Mysterious alien devices had been found inside a meteor that hit the North Pole. Inside the meteor, three of these devices were discovered. They were nearly identical, long, cylindrical objects and made a strange, ticking sound. It was found that the frequency of the ticking sound depended on the orientation of the rod. Astronomers soon realized that when the device was pointed along the plane of the Milky Way, the ticking was fastest, and when pointed outside this plane, the ticking nearly disappeared. This was a remarkable discovery, because it seemed to verify an age-old theory of the so-called ξ-particles, which supposedly are originated in distant stars but do not interact with ordinary matter. Apparently these alien devices are built of some material which the ξ-particles can "feel".

Soon an experiment was set up to measure the intensity of the ξ-particle flux on the surface of the Earth. The alien devices were pointed to different directions, in the plane of the Milky Way, and each of them scanned a sector of the plane, approximately 120 degrees wide.

The devices were operated for some time, and sets of measurement data were obtained. The data was tabulated and written into the files `data1.txt`,.... In these files the first column is the angle, measured from some arbitrary starting point, and the second column is the frequency of the ticks (ticks/second). The data files can be found in the webpage of this course, in the directory `pw2_datafiles`.

The First Task

Your first task is to construct an interpolating function to the measurement data to get a profile of the intensity of the ξ-particle flux on the surface of the Earth. Before the interpolation can reliably be made, you must calculate scaling factors for the three different data sets, which depend on the geometry of the devices, which, as mentioned, weren't precisely identical. The length and the diameter of the devices were measured multiple times with high-precision measuring instruments, and the results of the measurement (in centimeters) are tabulated in the files `diameter1.txt`, `length1.txt`,.... You must calculate the areas of the bases of the devices and their volumes, together with error estimates of both.

Then, do the interpolation, and divide the values of the resulting functions by the base areas of the correspondings cylinders. The sectors from which the devices gather data overlap somewhat, so when the results of the measurements are put together, the overlap must be eliminated. Unfortunately, it is not readily known, where exactly do they overlap, so you must compare the results of the interpolation and figure it out. Then construct a single, continuous interpolating function which is defined over the interval $0 \ldots 2\pi$.

The Second Task

There is a controversial theory of the effect which ξ-particles have on humans and here we're hoping to shed some light on the question. Your task is to calculate the total number of ξ-particles colliding with the Earth in one year. After you have divided the number of ticks per second by the base area of the detector, you can integrate this function over the angle to get the average intensity of the ξ-particle flux incident on the surface of the Earth (that is, the number of particles per unit area per unit time). This can be done either integrating the interpolating function, or using the add-on function `ListIntegrate`. The rest should be straightforward (we're assuming that in the course of one year, the Earth goes through all possible orientations with respect to the plane of the Milky Way).

The Third Task

Next, let's try to fit a function to the experimental data. As you can see when plotting your interpolating function, the ξ-particle flux intensity has rather sharp peaks for some values of the angle theta. Some common functions which could be used to describe these peaks are

- The Gaussian distribution:

$$G(x) = g_h e^{-g_w (x - x_g)^2}$$

- The Lorenzian distribution:

$$L(x) = \frac{l_h}{1 + l_w (x - x_l)^2}$$

where g_h and l_h determine the height of the peak, g_w and l_w determine the width of the peak and x_g and x_l give the position of the peak. The procedure of the fitting is described in the next paragraph for the Gaussian distribution. You must however make the fit for both types of peaks.

You can find the peak positions x_g simply by looking at the graph of the interpolating function. The number of the peaks determines the number of fitting functions that you will need. The function `Regress` will find you optimal values of the parameter g_h, but the parameter g_w, which you can assume to have the same value for all the fitting functions, must at first be guessed. Then you must refine your guess by constructing a table of the sums of the standard errors of the coefficients g_h for different values of g_w. Find the value of g_w (with the precision 0,1) which gives the minimum value for the sum of the standard errors.

Practical Issues

Carry out the above tasks and do *all* of your calculations with *Mathematica*.

Write a work report, which includes the following:

- (on the first page) your name and e-mail address, practice work number and date of return
- the assignment, briefly in your own words
- your throughly commented calculations
- a separate section for the results of the calculations

  First task The areas and volumes of the cylinders, including error estimates, and a plot of the interpolating function.

  Second task The total number of particles and the value of the average intensity. Note that this value will depend slightly on the way you have truncated the lists.

  Third task The fitted function in the above-mentioned form, using both the Gaussian and the Lorentzian lineshapes. Also, state the values of g_w and l_w which give the minimum error, in the sense described above.

Return the report into the IN-box in the theoretical physics lobby. If you experience problems, help is available every Tuesday at 8–11 in classroom KO130. If you can't come there, I can guide you be e-mail to some extent. My e-mail address is `jmattas@mail.student.oulu.fi`.

I will upload the answers (not the solutions) shortly.