Excercise 2

The Mathematica Book could be found from

http://documents.wolfram.com/mathematica/book/section-1

1. Lists, vectors, matrices. Read sections 1.8.1–4. and 2.5.5.

    a) Construct a table, which contains squares of first 100 positive integer. Calculate sum of the elements of that table.

    b) Define vectors $\mathbf{a} = 2\mathbf{i} - 3\mathbf{j}$ and $\mathbf{b} = -\mathbf{i} + \mathbf{j}$, and calculate the angle between them.

    c) Let $\mathbf{a} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = -\mathbf{j} - \mathbf{k}$. Find the unit vector, which is perpendicular to the plane spanned by vectors $\mathbf{a}$ and $\mathbf{b}$.

    d) Define matrix
    $$M = \left(\begin{array}{cc} 2 & -1 \\ 3 & 1 \end{array} \right)$$

    Find M:s inverse M^{-1}, determinant, eigenvalues and eigenvectors X_1 ja X_2. Check the results by calculating products MM^{-1}, MX_1 ja MX_2.

    e) This exercise associates with section 2.5.5. Construct some 3-dimensional matrix A, calculate its eigenvectors. Name the eigenvectors `a[i]`, $i = 1 \ldots 3$. and calculate all possible dot products `a[i].a[j]`. You can also try to index those eigenvectors by eigenvalues i.e. $a[\lambda_i]$

2. Plotting. Read sections 1.9.1–6 ja 1.9.8–9. Skip those containing `NDSolve-function` (it's for solving differential equations)

    a) Plot the graphs of $\sin x$ and $\cos 2x$, when $0 < x < 2\pi$, into same picture.

    b) Make a table of points $(n, n\text{:th prime number})$ when $n = 1, \ldots, 20$. (Use `Prime`-function). Plot points using `ListPlot`-function.

    c) Some object has a trajectory which x and y-coordinates obey functions $x(t) = t^2 + 5t - 1$ and $y(t) = 10\sin(1/(t^2 + 10))$. Plot the trajectory, when $-2 < t < 2$.

    d) Fiqure out what `Manipulate`-function does. Test it with for example `Plot`-function.

3. Using files: Read 1.11.1–3. (In a newer version of Mathematica the command `!!` has been replaced with `FilePrint`. $n!! = n(n-2)(n-4) \cdots 3 \cdot 1$, jos n on pariton, ja $n!! = n(n-2) \cdots 4 \cdot 2$, jos n on parillinen.

a) Write the table you made in a previous excercise (b) to a file so that it contains two colmuns. Read that table again from the file. Write same table to the file by using <<-command. Open file with some text editor and write `a=` before the table (save). Run the file from Mathematica. How you can append someting to the file?

b) Let the velocity of an object be

$$\mathbf{v} = (2\mathbf{i} - \mathbf{j} + \frac{1}{2}\mathbf{k})\text{m/s}.$$

When $t = 0$ the object is at

$$\mathbf{r}(0) = (2\mathbf{k} - 6\mathbf{i})\text{m}.$$

Calculate the location of the object $\mathbf{r}(t)$ at $t = 3\text{s}$

Define a function that describes the trajectory of the object in function of time. Plot the trajectory using `ParametricPlot3D`, when $0 < t < 10\text{s}$.

Write all the commads in a separate file with file extension `.m`. Run the file from Mathematica:

`<<"<path>/file.m"`

c) Try previous with velocity that depends on time. You will need integration: `Integrate[f,{x,xmin,xmax}]`, see. sec. 1.5.3.

$$\mathbf{v}(t) = (2at\mathbf{i} - \mathbf{j} + \frac{bt^2}{2}\mathbf{k})\text{m/s}.$$

where $a = 0.1\text{s}^{-1}$ ja $b = 10^{-4}\text{s}^{-2}$.