Excercise 4

1. Read section 3.8.3. Focus especially on functions

   ```
   Interpolation
   InterpolatingPolynomial
   ```

   Check also examples from help pages of those functions

2. Make table of points $(x, sin(x))$, $-2\pi < x < 2\pi$, with 0.5 stepping. Define three interpolating functions (orders of 0,1, and 2) and plot them together with $\sin(x)$.

3. Make a table ofpoints $(x_i, f(x_i))$, where

   $$f(x) = \frac{x}{1 + x\sin^2 x}$$

   and $0 < x_i < 3\pi$; $i = 1, \ldots, n$; $n = 2, 5, 10, 20, 50$. Plot $f(x)$ and points in same graph.

   Write a function, which calculates polynomial

   $$p(x) = \sum_{i=1}^{n} f(x_i) \prod_{j=1, j\neq i}^{n} \frac{x - x_j}{x_i - x_j}$$

   Check that $p(x_i) = f(x_i)$. Plot $p(x)$ with $n = 2, 5, 10, 20, 50$.

   Repeat same task (find that $p(x_i) = f(x_i)$ and plotting) with build-in function

   ```
   InterpolatingPolynomial[data,x]
   ```

   Notice that interpolating polynomial is definitely not suitable for extrapolation.

4. Use same table of points created in previous excersice. Make a piecewise defined interpolating function, where q_i:s are 1st order polynomial: $q_i(x) = a_i(x - x_i) + b_i$, if $x_i < x < x_{i+1}$. Coefficients a_i ja b_i come from equations

   $$q_i(x_i) = b_i = f_i$$
   $$q_i(x_{i+1}) = a_i(x_{i+1} - x_i) + b_i = f_{i+1}$$

   Defining a piecewise defined functions could be done with `If`- and `For`-expressions.