Excercise 6

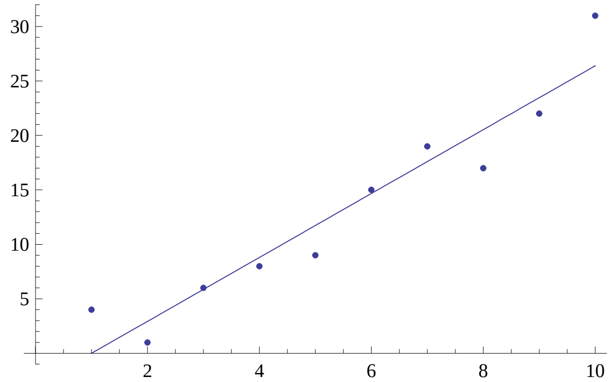
1. Go through helps, syntax and examples of functions `Fit`, `Dt` and `Regress`.
   note: `Regress` needs `LinearRegression`-package.

   Fitting a function to the data

   - Motivation: Calculate parameters of some theoretical model from the data.
   - Commoly used method – *Least square fitting*
     Data from some measurement is presented in form (x_i, y_i), $i = 1, \ldots, n$. We find function f so that sum
     $$\sum_{i=1}^{n}(f(x_i) - y_i)^2$$
     gets it minimum value.
   - For example if $f(x) = ax + b$: sum becomes
     $$S = \sum_{i=1}^{n}(ax_i + b - y_i)^2.$$
     The coefficients a and b are (take derivatives with respect to a and b, set them to zero and solve a and b).
     $$a = \frac{nS_{xy} - S_x S_y}{nS_{xx} - S_x^2}$$
     $$b = \frac{S_y S_{xx} - S_x S_{xy}}{nS_{xx} - S_x^2},$$
     where $S_x = \sum x_i$, $S_y = \sum y_i$, $S_{xx} = \sum x_i^2$ and $S_{xy} = \sum x_i y_i$.

```
In[629]:= a = Table[{n, Prime[n] + Random[Integer, {-2, 2}]}, {n, 1, 10}];
          y = Fit[a, {1, x}, x];
          Show[ListPlot[a], Plot[y, {x, 1, 10}]]
```

Out[631]=



- The function f could be some other form (e.g. 2nd order polynomial etc.) That naturally depends on the theory.

2. You have measured electric current as a function of voltage over some mystical component. File `h06_data.txt` contains measured data-points (V_i, I_i). Those points (should) obey Ohm's law

$$V = RI. \qquad (1)$$

   Fit function of form (**??**) to the data points using least square fitting-method (presented above: calculate sums etc.) and calculate the resistance R of the component. Compare your results to the results from built-in function (`Fit[]`).

3. Estimate the error of the resistance you have measured. Use `Regress` to the data points. You get table of statistics. `SE` gives you Standard Error. Calculate the values of resistance $R_i = V_i/I_i$ and from those average value

$$\overline{R} = \frac{1}{n}\sum_{i=0}^{n} R_i,$$

   and standard deviation

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=0}^{n}(R_i - \overline{R})^2}$$

4. Error of the resistance could be estimated using total differenitial:

$$\begin{aligned} dR &= \frac{\partial R}{\partial V}dV + \frac{\partial R}{\partial I}dI \\ &= \frac{dV}{I} - \frac{V}{I^2}dI \end{aligned}$$

   This equation is approximately valid for finite change

$$\Delta R = \frac{\Delta V}{I} - \frac{V}{I^2}\Delta I$$

Let V and I be the average values of voltage and current. We denote error of the measurement by ΔV and ΔI (e.g. maximum differcence from mean). We get the upper limit of the error of the resistance

$$\Delta R \leq |\frac{\Delta V}{I}| + |\frac{V}{I^2}\Delta I|$$

Calculate the error estmation for measurement stored in `h06_data2.txt` with total differetial –method. (*note:* this method is valid only if all measuremets are independent and done with approximately same initial conditions).