Excercise 8

1. **Harmonic Oscillator**
   Driven and damped harmonic oscillator obeys differential equation

$$x''(t) + \gamma x'(t) + \omega^2 x(t) = f(t) \tag{1}$$

   Let $f(t) = f_0 \cos \omega_d t$. Solve equation (1). Illustrate and try to explain the solutions in following situations. Plot also x(t) vs. x'(t) using `ParametricPlot`.

   a) $x(0) = 1$, $x'(0) = 0$, $f_0 = 0$, $\gamma = 0$ (simple harmonic oscillator)
   b) $x(0) = 1$, $x'(0) = 1$, $f_0 = 0$, $\gamma = 0.1$
   c) $x(0) = 1$, $x'(0) = 0$, $f_0 = 1$, $\omega = 1$, $\omega_d = 1$, $\gamma = 0.1$
   d) $x(0) = 1$, $x'(0) = 0$, $f_0 = 1$, $\omega = 1$, $\omega_d = 0.9$, $\gamma = 0.1$
   e) $x(0) = 0$, $x'(0) = 0$, $f_0 = 1$, $\omega = 1$, $\omega_d = 1.5$, $\gamma = 0.9$

   In c) d) and e) calculate the integral

$$\frac{1}{T} \int_{t_0}^{t_0+T} x'(t) f(t) dt \tag{2}$$

   It describes the average power transfered from driving force to the oscillator. Here t_0 is time you can choose freely and $T = 2\pi/\omega_d$.

2. **Predator-prey- or Lotka-Volterra-model**
   Two populations are competing for the same restricted resource of the environment. Growth of the populations is described by system of differential equations:

$$\frac{dN_1}{dt} = \epsilon_1 N_1(t) - \beta_1 N_1^2(t) - \gamma_1 N_1(t) N_2(t)$$
$$\frac{dN_2}{dt} = \epsilon_2 N_2(t) - \beta_2 N_2^2(t) - \gamma_2 N_2(t) N_1(t)$$

   Let's look more closely what the parameters are describing:

   - ϵ_i – how fast the population N_i would grow, if the environment doesn't restrict the growth.
   - β_i – how much the availability of the resources restrict the growth of population N_i.
   - γ_i – How much the presense of the other population restricts the growth of the population N_i.

Solve the system of equations and illustrate following situations:

a) $\beta_i = \gamma_i = 0$, $\epsilon_1 = 0.3$, $\epsilon_2 = 0.4$, $N_i(0) = 1$.

b) $\beta_i = 0.2$, $\gamma_i = 0$, $\epsilon_1 = 0.3$, $\epsilon_2 = 0.4$, $N_i(0) = 1$.

c) $\beta_i = 0.2$, $\gamma_i = 0.1$, $\epsilon_1 = 0.3$, $\epsilon_2 = 0.4$, $N_i(0) = 1$.

d) $\beta_i = 0$, $\gamma_1 = 0.1$, $\gamma_2 = -0.15$, $\epsilon_1 = 0.3$, $\epsilon_2 = -0.4$, $N_i(0) = 1$.

Explain the solutions.