Excercise 10

Finite difference method

- Finite difference method is a common method for solving differential equations numerically.
- In this method the derivatives in the differential equation are replaced with the approximations of the derivatives calculated from the Taylor-series. (like we did in Exercise 5).
- Approximations:
  - 1st derivative
    $$f'(x_i) = \frac{f_{i+1} - f_{i-1}}{2h} - \frac{1}{6} f'''(x_i) h^2 + \dots \tag{1}$$
  - 2nd derivative
    $$f''(x_i) = \frac{f_{i+1} - 2f_i + f_{i-1}}{h^2} - \frac{1}{12} f^{(4)}(x_i) h^2 - \dots \tag{2}$$
  - 3rd derivative
    $$f'''(x_i) = \frac{f_{i+2} - 2f_{i+1} + 2f_{i-1} - f_{i-2}}{2h^2} - \frac{1}{4} f^{(5)}(x_i) h^2 + \dots \tag{3}$$
- After replacement the values of the solution could be calculated from the recursive formula.

1. Let's revisit the projectile motion problem. The differential equations of the motion are
   $$F_y = my''(t) = mg - ky'(t)$$
   $$F_x = mx''(t) = -kx'(t)$$
   where $g = -9.81$ and $k = 0.1$. Solve using finite difference method and plot the trajectory of the projectile ($m = 0.5$ kg) with initial conditions $y(0) = x(0) = 0$ and $x'(0) = 10$, $y'(0) = 20$.
   Note: You need the value of function in first two points. Approximate derivative
   $$x'(t_1) \approx \frac{x_2 - x_1}{h}$$
   and solve the x_2 from that.

2. Solve differential equation

$$y''(x) + 4xy'(x) + (102 + 4x^2)y(x) = 0 \tag{4}$$

with Mathematica's built-in functions. Initial conditions are $y(0) = 0$ ja $y'(0) = 1$. `DSolve` will give you a complex function. Use

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ComplexExpand
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to calculate the complex exponential functions and you will find that the solution is real.

3. Solve the differential equation (4) by using *finite differential method* when $-3 < x < 3$. (i.e. positive and negative direction). Initial conditions are $y(0) = 0$ and $y'(0) = 1$, and numbers of evaluation points $n \approx 1000$. Try to reduce n. What happens the the solution?