

Harjoitus 10 -- ratkaisut

1

```
Clear[m, g, k, h]

Solve[m * (fp1 - 2 * f + fm1) / h^2 == m * g - k * (fp1 - fm1) / (2 * h), fp1]


$$\left\{ \left\{ fp1 \rightarrow \frac{fm1 h k + 4 f m - 2 fm1 m + 2 g h^2 m}{h k + 2 m} \right\} \right\}$$


Remove[x, y]

eq1 = m * y''[t] == m * g - k * y'[t]
eq2 = m * x''[t] == -k * x'[t]

m y''[t] == g m - k y'[t]

m x''[t] == -k x'[t]

n = 1000; t[0] = 0.0; t[n] = 5; h = (t[n] - t[0]) / (n - 1);

Do[t[i] = t[0] + i * h, {i, 1, n}];

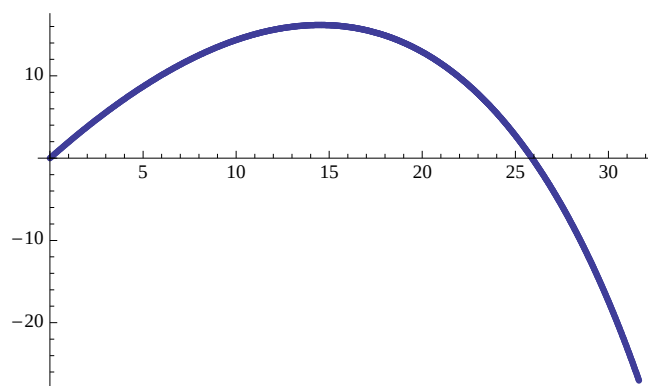
y[0] = 0; y[1] = y[0] + h * 20;

x[0] = 0; x[1] = x[0] + h * 10;

m = 0.5; g = -9.81; k = 0.1;

Do[
  y[i] =
    (h * k - 2 * m) / (h * k + 2 * m) * y[i - 2] + 4 * m / (h * k + 2 * m) * y[i - 1] + 2 * g * m * h^2 / (h * k + 2 * m);
  x[i] = (h * k - 2 * m) / (h * k + 2 * m) * x[i - 2] + 4 * m / (h * k + 2 * m) * x[i - 1];, {i, 2, n}]

ListPlot[Table[{x[i], y[i]}, {i, 0, n}], PlotRange -> All]
```



2

```
eq = y''[x] + 4 x y'[x] + (102 + 4 x^2) * y[x] == 0
r[x_] = y[x] /. Flatten[DSolve[{eq, y[0] == 0, y'[0] == 1}, y[x], x]]
```

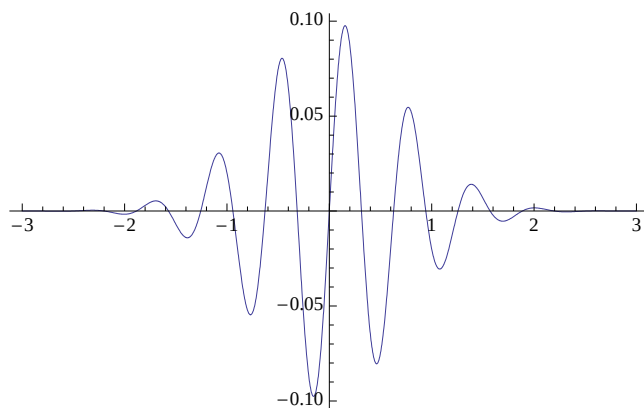
$$(102 + 4 x^2) y[x] + 4 x y'[x] + y''[x] == 0$$

$$-\frac{1}{20} i e^{-x(10 i + x)} (-1 + e^{20 i x})$$

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r[x_] = ComplexExpand[r[x]] // Simplify
```

$$\frac{1}{10} e^{-x^2} \sin[10 x]$$

```
Plot[r[x], {x, -3, 3}, PlotRange -> All]
```



3.

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Remove[x, y]
```

```
n = 100; x[0] = 0.0; x[n] = 3.0; x[-n] = -3.0; h = (x[n] - x[0]) / (n - 1);
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Do[x[i] = x[0] + i * h; x[-i] = x[0] - i * h;, {i, 1, n - 1}]
```

```
y0 = 0; dy0 = 1;
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```
y[0] = y0;
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```
y[1] = y[0] + dy0 * h;
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```
y[-1] = y[0] - dy0 * h;
```

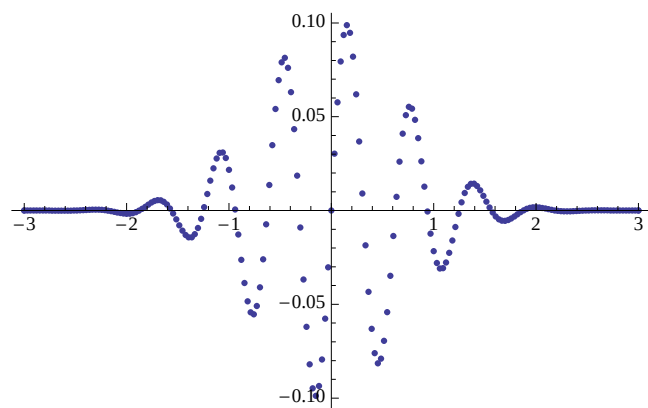
```
Do[y[i] = 1 / (1 / h^2 + 2 * x[i - 1] / h) *
```

```
(y[i - 1] * (2 / h^2 - 102 - 4 * x[i - 1]^2) + y[i - 2] * (-1 / h^2 + 2 * x[i - 1] / h));
```

```
y[-i] = 1 / (1 / h^2 - 2 * x[-i + 1] / h) *
```

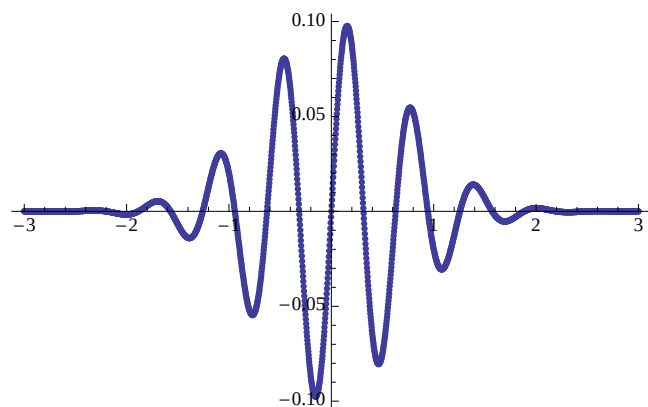
```
(y[-i + 1] * (2 / h^2 - 102 - 4 * x[-i + 1]^2) - y[-i + 2] * (1 / h^2 + 2 * x[-i + 1] / h));, {i, 2, n}]
```

```
ListPlot[Table[{x[i], y[i]}, {i, -n, n}], PlotRange → All]
```

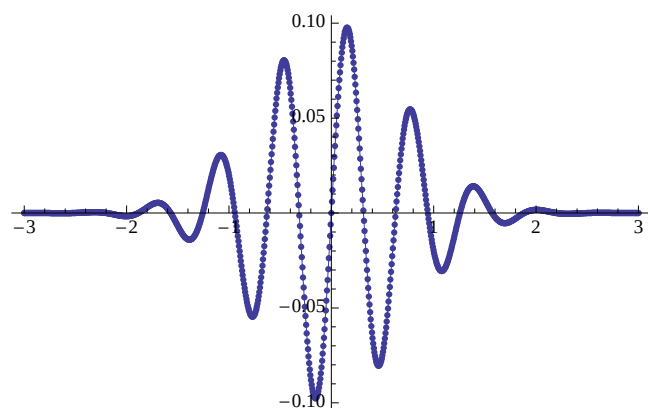


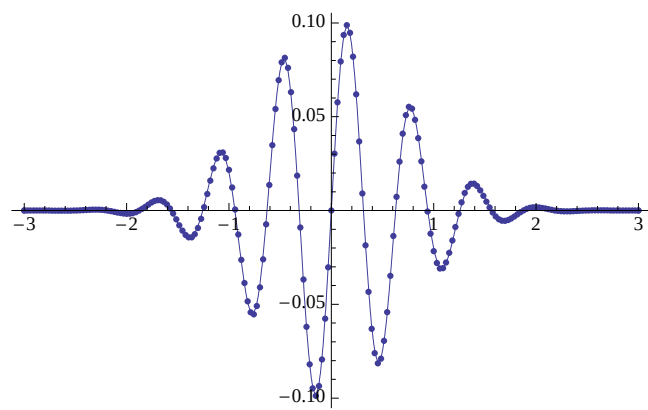
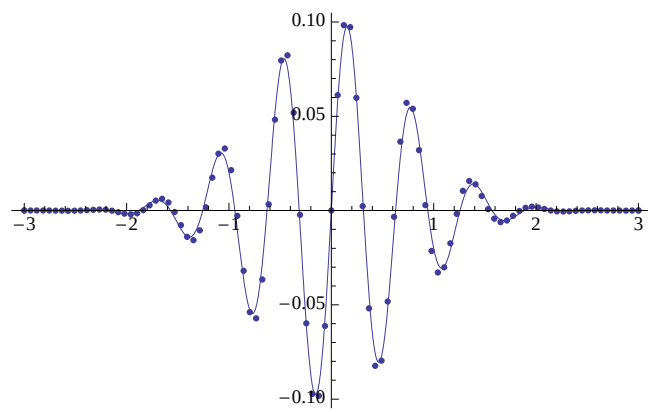
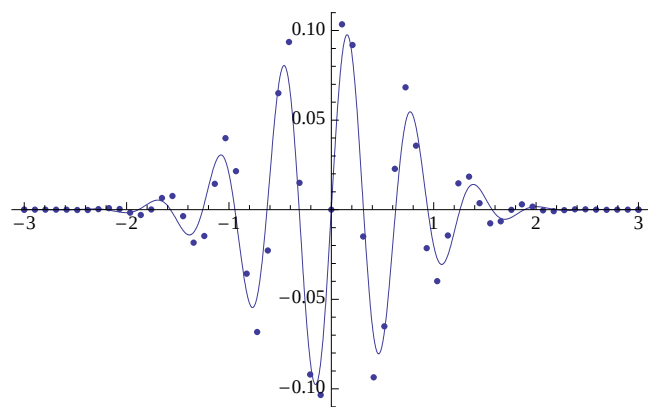
```
exa[m_] := (
  Clear[x, y];
  n = m;
  x[0] = 0.0; x[n] = 3.0; x[-n] = -3.0; h = (x[n] - x[0]) / (n - 1);
  Do[x[i] = x[0] + i * h; x[-i] = x[0] - i * h; , {i, 1, n - 1}];
  y0 = 0; dy0 = 1; y[0] = y0; y[1] = y[0] + dy0 * h; y[-1] = y[0] - dy0 * h;
  Do[y[i] = 1 / (1 / h^2 + 2 * x[i - 1] / h) *
    (y[i - 1] * (2 / h^2 - 102 - 4 * x[i - 1]^2) + y[i - 2] * (-1 / h^2 + 2 * x[i - 1] / h));
    y[-i] = 1 / (1 / h^2 - 2 * x[-i + 1] / h) * (y[-i + 1] * (2 / h^2 - 102 - 4 * x[-i + 1]^2) -
      y[-i + 2] * (1 / h^2 + 2 * x[-i + 1] / h)); , {i, 2, n}];
  Show[ListPlot[Table[{x[i], y[i]}, {i, -n, n}], PlotRange → All],
    Plot[r[t], {t, -3, 3}, PlotRange → All]]
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exa[1000]
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exa[500]
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exa[100]**exa[50]****exa[30]**

exa[20]

