# Excercise 11

Go through following excercises with the solution file `h11_sol.nb`. Comment every step/line in the solution.

1. Let's look at the object that feels the gravity of the Earth (a bit larger and scale than before). Equation of motion is

$$m\mathbf{r}''(t) = -G\frac{mM}{|\mathbf{r}(t)|^3}\mathbf{r}(t) - \gamma|\mathbf{r}'(t)|\mathbf{r}'(t). \tag{1}$$

and in two dimensions $\mathbf{r}(t) = x(t)\hat{\mathbf{x}} + y(t)\hat{\mathbf{y}}$ ($\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ are unit vectors) , $|\mathbf{r}(t)| = \sqrt{x(t)^2 + y(t)^2}$ is the distance between object and center of the Earth and $M$ is mass of the Earth, G is the gravitational constant. Equation (1) corresponds system of two non-linear differential equations

$$\begin{aligned} x''(t) &= -\frac{GM}{(x(t)^2+y(t)^2)^{3/2}}x(t) - \frac{\gamma}{m}\sqrt{x'(t)^2+y'(t)^2}\,x'(t) \\ y''(t) &= -\frac{GM}{(x(t)^2+y(t)^2)^{3/2}}y(t) - \frac{\gamma}{m}\sqrt{x'(t)^2+y'(t)^2}\,y'(t) \end{aligned}$$

a) Construct functions $\mathbf{r}(0)$, which lies in somewhere on the surface of the sphere, radius $R$ (in xy-plane, on the circle, radius $R$ ) and $\mathbf{r}'(0)$ at $\mathbf{r}(0)$ (direction given in respect to surface of the Earth).
Solve equations of motion using `NDSolve`, when $\gamma = 0$. Plot $(x(t), y(t))$. Try different values for intial location and velocity. With which values the object lands to the initial location?

b) Add the term of air resistance (i.e. $\gamma \neq 0$) and again try to find initial values so that object lands to the $\mathbf{r}(0)$

c) Solve the equation on motion using *finite difference method.* Now equatios are so complicated that $x_{i+1}$:tä and $y_{i+1}$ can't be solved directly. The solution could be obtained using `FindRoot`-function, or you can make little code that uses bisection or Newton's method. With `FindRoot` the procedure is

```
Do[{x[i + 1], y[i + 1]} = {x[i + 1], y[i + 1]} /.
   FindRoot[{(x[i + 1] - 2 x[i] + x[i - 1])/h^2 + G M x[i]/(x[i]^2 + y[i]^2)^(3/2) == 0,
     (y[i + 1] - 2 y[i] + y[i - 1])/h^2 + G M y[i]/(x[i]^2 + y[i]^2)^(3/2) == 0},
    {{x[i + 1], x[i]}, {y[i + 1], y[i]}}], {i, 2, n}] // Quiet
```