Excercise 12

1. Equation of standing wave motion with boundary conditions $y(0) = 0$ ja $y(L) = 0$ is an example of differential equation that has an eigenvalue:

$$y''(x) + k^2 y(x) = 0 \tag{1}$$

In general, only solution to this kind of problem is $y(x) = 0\ \forall x$. Also mathemaca gives you only that solution

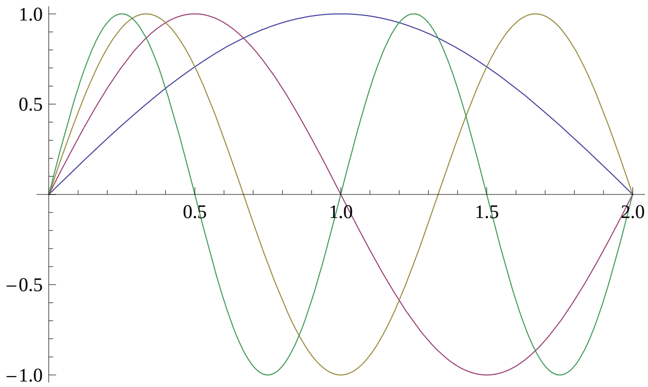
```
In[814]:= DSolve[{y''[x] + k^2 y[x] == 0, y[0] == 0, y[L] == 0}, y[x], x]

Out[814]= {{y[x] → 0}}
```

There are non-trivial solutions for equation (1), if k has some certain value. Those k's are the eigenvalues of the equation (boundary value problem).

```
In[820]:= Plot[Evaluate[Table[Sin[k x] /. k → i Pi/2, {i, 1, 4}]], {x, 0, 2}]
```

Out[820]=



Non-trivial solutions could be reached, when solving the equation in two parts:

- $x \in [0, L/2]$, with boundary conditions $y(0) = 0$, $y'(0) = 1$
- $x \in [L/2, L]$, with boundary conditions $y(L) = 0$, $y'(L) = \pm 1$

and joining those solutions at $x = L/2$.

Solve this problem using following procedure: (you can use `(N)DSolve` or finite difference method.)

two point boundary value problems

(a) Divide the range of the solution in two parts: $x \in [x_{lower}, x_i]$ and $x \in [x_i, x_{upper}]$

(b) Guess the first eigenvalue

(c) Solve differential equations in both areas with given boundary conditions: $y_1(x)$, $y_2(x)$

(d) Calculate Δ (from condition: $y(x)$ continuous and differentiable at $x = x_i$)

$$\frac{y_1'(x_i)}{y_1(x_i)} = \frac{y_2'(x_i)}{y_2(x_i)} \Rightarrow \Delta = \frac{y_1'(x_i)}{y_1(x_i)} - \frac{y_2'(x_i)}{y_2(x_i)} \tag{2}$$

(e) Repeat (b), (c) and (d) to find eq. (2) of opposite sign. (if opposite signs, there must be $\Delta = 0$, with some eigenvalue between first and second guess)

(f) Find that eigenvalue using FindRoot or writing e.g. Bisection method algorithm.

(g) Repeat from (b) to find other eigenvalues.