ATKIV Numerical Programming
Exercise 3.3 Neville's algorithm

J. Isohätälä

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Let (x_i, y_i) be the points to be interpolated by a polynomial, $i = 1, \ldots, n$. Denote polynomial interpolating points (x_i, y_i), $(x_{i+1}, y_{i+1}), \ldots, (x_k, y_k)$ by $P_{i,\ldots,k}(x)$. We have the recurrence relation

$$P_{i,\ldots,i+m}(x) = \frac{(x - x_{i+m})P_{i,\ldots,i+m-1}(x) + (x_i - x)P_{i+1,\ldots,i+m}(x)}{x_i - x_{i+m}}. \tag{1}$$

We need the polynomial P_{1234}, so let's start from that. From Eq. (1) we get

$$P_{1234} = \frac{(x-4)P_{123} + (1-x)P_{234}}{1-4} \tag{2}$$

In order to proceed, we need P_{123} and P_{234}. Again, Eq. (1) gives

$$P_{123} = \frac{(x-3)P_{12} + (1-x)P_{23}}{1-3} \tag{3}$$

We must go level by level, until we can write the polynomial. Remember, that $y_i = P_i$.

$$P_{12} = \frac{(x-2) + (1-x)3}{1-2} = 2x - 1 \tag{4}$$

It's a good idea to always check that the resulting polynomial does pass through the points it's supposed to.

$$P_{23} = \frac{(x-3)3 + (2-x)6}{2-3} = 3x - 3 \tag{5}$$

Now, substituting polynomials from Eqs. (4), (5) to (3) we get

$$P_{123} = -\frac{1}{2}\left[(x-3)(2x-1) + (1-x)(3x-3)\right] = \frac{x}{2} + \frac{x^2}{2} \tag{6}$$

Then,

$$P_{234} = \frac{(x-4)P_{23} + (2-x)P_{34}}{2-4}, \tag{7}$$

$$P_{34} = \frac{(x-4)6 + (3-x)8}{-1} = 2x. \tag{8}$$

From Eqs. (5) and (8) we have

$$P_{234} = -\frac{1}{2}(3x-3)(x-4) - x(2-x) = -6 + \frac{11x}{2} - \frac{x^2}{2}. \tag{9}$$

Finally, we may write P_{1234} from Eq. (2)

$$P_{1234} = \ldots = 2 - \frac{19}{6}x + \frac{5}{2}x^2 - \frac{1}{3}x^3. \tag{10}$$

Finally,

$$P_{1234}(5/2) = 9/2. \tag{11}$$