1. Which of the following processes are allowed? If not, why?

$$\begin{aligned} n &\to p + e^- + \nu_e \\ n + n &\to \pi^+ + \pi^- + \pi^0 \\ n &\to p + \pi^- \\ e^- + \nu_\mu &\to \mu^- + \nu_e \\ \tau &\to \mu + \bar{\nu}_\mu + \nu_e \end{aligned}$$

2. Consider the reaction $p + p \to p + p + p + \bar{p}$.

   a) In LHC the protons collide head-on with the same kinetic energy. What is the minimum kinetic energy (for each proton) required for the process above?

   b) In a fixed target experiment one of the original protons is at rest. What is the minimum kinetic energy of the moving proton in this case?

3. The Dirac γ-matrices obey the relations

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}, \quad \gamma^5 \equiv \gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3, \quad (\gamma^\mu)^\dagger = \gamma^0\gamma^\mu\gamma^0,$$

   (where the last property is valid in the "standard" representation used in the lectures). Show that

   a) $\mathrm{Tr}\,[\gamma^\mu] = 0$

   b) $\{\gamma_\mu, \gamma_5\} = 0$

   c) $\gamma_5^2 = 1$

   d) $\gamma_5^\dagger = \gamma_5$

4. Let us consider lowest-order elastic QED scatterings

$$e^- + \mu^- \to e^- + \mu^- \quad \text{and} \quad e^+ + \mu^- \to e^+ + \mu^-$$

   Draw the Feynman diagrams. Using the Feynman rules, write down the amplitudes $\mathcal{M}$ in terms of the fermion functions $u, v, \bar{u}, \bar{v}$ (these are functions of momenta p_i and spin s_i).

5. Evaluate the spin-averaged square of the amplitude $\langle|\mathcal{M}|^2\rangle$ for the $e^+ + \mu^- \to e^+ + \mu^-$ -scattering in excercise 4 in the ultrarelativistic limit (i.e. you can set $m_e = m_\mu = 0$).

   Note:

$$\mathrm{Tr}\,[\gamma^\mu \not{a} \gamma^\nu \not{b}] = 4[a^\mu b^\nu + a^\nu b^\mu - g^{\mu\nu} a \cdot b]$$

$$\sum_s u(p,s)\bar{u}(p,s) = \not{p} + m \qquad \sum_s v(p,s)\bar{v}(p,s) = \not{p} - m.$$

$p \sim (uud)$	$m_p \approx 938.280\mathrm{MeV}$	$m_e \approx 0.511\mathrm{MeV}$
$n \sim (udd)$	$m_n \approx 939.573\mathrm{MeV}$	$m_\mu \approx 105.659\mathrm{MeV}$
$\pi^+ \sim (u\bar{d})$	$m_{\pi^+} \approx 139.563\mathrm{MeV}$	$m_\tau \approx 1784\mathrm{MeV}$
$\pi^0 \sim (u\bar{u} - d\bar{d})$	$m_{\pi^0} \approx 134.964\mathrm{MeV}$	$m_{\nu_i} \approx 0$