1. Using $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$ show that

   a) $\mathrm{Tr}\,\gamma^\mu\gamma^\nu = 4g^{\mu\nu}$

   b) $\mathrm{Tr}\,\gamma^\mu\gamma^\nu\gamma^\rho = 0$

   c) $\mathrm{Tr}\,\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma = 4(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho})$

   d) $\gamma^\mu \not{a} \gamma_\mu = -2\not{a}$

2. Consider elastic $e^- + \mu^- \to e^- + \mu^-$ -scattering at the ultrarelativistic limit, where you can set $m_e = m_\mu = 0$.

   a) Starting from the result at page 137 in the notes, find the center of mass frame averaged amplitude $\langle|\mathcal{M}|^2\rangle$.

   b) Find the center of mass frame differential cross-section. Let $E$ be the electron energy and $\theta$ the scattering angle.

$$\left[\textit{Result:} \qquad \frac{d\sigma}{d\Omega} = \frac{1}{(8\pi)^2}\frac{e^4}{2E^2}\left(\frac{1+\cos^4\frac{\theta}{2}}{\sin^4\frac{\theta}{2}}\right)\right]$$

3. Let us consider the lowest order elastic $e^- + e^+ \to e^- + e^+$ -scattering (Bhabha scattering).

   a) Write the amplitude $\mathcal{M}$ in terms of $u$, $\bar{u}$, $v$, $\bar{v}$. There are 2 diagrams, thus, $\mathcal{M} = \mathcal{M}_1 + \mathcal{M}_2$.

   b) Average over initial spins and sum over final spins and express

$$\langle|\mathcal{M}|^2\rangle = \langle|\mathcal{M}_1|^2\rangle + \langle|\mathcal{M}_2|^2\rangle + 2\langle|\mathcal{M}_1\mathcal{M}_2^*|\rangle$$

   in terms of traces over $\gamma$-matrices. Note that here you have to use the completeness relations

$$\sum_s u(p,s)\bar{u}(p,s) = \not{p} + m \qquad\qquad \sum_s v(p,s)\bar{v}(p,s) = \not{p} - m.$$