1. Show that (by expanding in Taylor series)

$$e^{i\theta\sigma_1} = 1\cos\theta + i\sigma_1 \sin\theta\,.$$

Convince yourself that similar result also holds for σ_2 and σ_3. Show then

$$e^{i\theta_k\sigma_k} = 1\cos|\theta| + i\hat{\theta}_k\sigma_k \sin|\theta|\,,$$

where $|\theta| = \sqrt{\sum_i \theta_i^2}$ and $\hat{\theta}_k = \theta_k/|\theta|$. (Hint: show that $(\hat{\theta}_k\sigma_k)^2 = 1$.)

2. Using the operators

$$J_\pm = J_x \pm iJ_y$$

and the relations

$$J_\pm|jm\rangle = \sqrt{j(j+1) - m(m\pm 1)}|j(m\pm 1)\rangle, \qquad J_z|jm\rangle = m|jm\rangle\,,$$

construct the spin $j = 1$ generator matrices

$$(\lambda_k)_{m'm} = \langle 1m'|J_k|1m\rangle.$$

Show that this is consistent with the identification

$$|11\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad |10\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad |1-1\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix},$$

or λ_z, $\lambda_\pm$ operate on these vectors as they should.

3. In a *collider* experiment 2 protons move to opposite directions with the same speed and collide head-on; in a *fixed target* experiment the other proton is at rest (as seen in the laboratory frame). Let us assume that the total energy of the protons in the collider experiment is $2T$ (T: beam energy). What should the energy of the moving proton be in the fixed target experiment in order for the center-of-mass energy to be equal to the collider experiment? Calculate this for $T = 10\,\text{GeV}$ and $T = 10\,\text{TeV}$. This shows the advantage of doing collider experiments.