1. Show that the γ_μ-matrices given at lectures obey algebra

$$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}$$

2. Because so few particles are eigenstates of C it is not very useful symmetry. However, we can construct so-called G-parity as a combination of C and isospin rotation

$$G = CI_2, \qquad I_2 = e^{-i\pi\sigma_2/2}$$

where σ_2 is a Pauli matrix. Thus, the isospin rotation transforms isospin doublets

$$\begin{pmatrix} u' \\ d' \end{pmatrix} = I_2 \begin{pmatrix} u \\ d \end{pmatrix} = \begin{pmatrix} -d \\ u \end{pmatrix},$$

and for conventionally defined antidoublet

$$\begin{pmatrix} -\bar{d}' \\ \bar{u}' \end{pmatrix} = I_2 \begin{pmatrix} -\bar{d} \\ \bar{u} \end{pmatrix} = - \begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix}.$$

Show that

$$G\pi = -\pi, \qquad G\eta = \eta$$

for any pion $+, -, 0$. (For quark contents see page 56 in lecture notes.) In fact, all mesons which carry no net strangeness, charm, ... are eigenstates of G. Because this is a symmetry of strong sector, *G-parity is conserved in strong interactions*, but not in electromagnetic or weak. G-parity is a multiplicative quantum number like parity; thus, a state with n pions has G-parity of $(-1)^n$.

3. Mass of η-meson is 549 MeV so that it has enough energy to decay strongly into 2π or 3π. However this does not happen:

a) Why decay $\eta \to 2\pi$ is forbidden in strong and electromagnetic interactions?

b) Why $\eta \to 3\pi$ is forbidden in strong but not in electomagnetic interactions (see previous question)?

This forbids strong decays of η, and it is relatively long-lived particle.