1. Show that the phase space integral can be written in the Lorentz-invariant form
$$\int \frac{d^3\mathbf{p}}{(2\pi)^3 2E_{\mathbf{p}}} = \int \frac{d^4p}{(2\pi)^4}\, 2\pi\, \delta(p^2 - m^2)\Theta(p^0)$$
where $p = (p^0, \mathbf{p})$. Thus, the standard phase space integral is "standard" integral over 4-momentum but restricted on-shell!

2. For two particles to interconvert, $A \leftrightarrow B$ (as K^0 and $\bar{K}^0$ do), it is necessary that they have the same mass (which in practice implies that they are antiparticles of each other), same electric charge and same baryon and lepton numbers. Show that, in the Standard Model, neutral mesons satisfy this condition. What is quark content of these particles? Why doesn't the neutron mix with the antineutron? Why it is impossible to see the mixing of neutral strange *vector* ($J = 1$) mesons K^*, $\bar{K}^*$? (Hint: consider decay channels and lifetime.)

3. Let us apply the result for the decay rate derived in page 111 in lecture notes to $\rho \to \pi\pi$ -decay. Let us take the particle masses $m_A = m_\rho = 770\,\mathrm{MeV}$, $m_1 = m_2 = m_\pi = 140\,\mathrm{MeV}$, and let us guess the amplitude $\mathcal{M} = 2\,\mathrm{GeV}$ (which is reasonable for hadron physics).

   a) What is the result for the lifetime of ρ? (The real lifetime of ρ is $4.4 \times 10^{-24}\,\mathrm{s}$.

   b) Sketch the decay rate as a function of $m_A = m_\rho$. At which m_A does this reach the maximum value (roughly)?