1. a) Show that the Mandelstam variables s, t, u associated with the scattering $A + B \to C + D$,

$$s = (p_A + p_B)^2, \qquad t = (p_A - p_C)^2, \qquad u = (p_A - p_D)^2$$

are not independent:

$$s + t + u = m_A^2 + m_B^2 + m_C^2 + m_D^2$$

b) Assume that $m_A = m_B = m_C = m_D \equiv m$. What is the kinematically allowed domain on s, t-plane?

2. If we assume that the incoming particle beam is along z-axis, we can define another useful kinematical variable, *rapidity* y, as

$$y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z}.$$

a) Show that the four-momentum can be written as $p = (m_T \cosh y, p_x, p_y, m_T \sinh y)$, where $m_T = \sqrt{m^2 + p_x^2 + p_y^2}$ is the "transverse mass".

b) Let us assume we have two particles A and B moving along z-axis with rapidities y_A and y_B. Using the velocity addition formula, show that the rapidity of B in the rest frame of A is $y = y_B - y_A$, i.e. rapidity is an additive quantity.

3. Following the example in the lecture notes, let us consider a scalar theory with 3 scalar fields and interaction $\hat{\mathcal{L}}_I = g\hat{\phi}_A\hat{\phi}_B\hat{\phi}_C$. Let us assume that $m_A > m_B + m_C$, so that the decay $A \to B + C$ becomes possible.

a) Draw the diagrams of order g, g^2, g^3 for decay $A \to B + C$

b) What is the amplitude $\mathcal{M}$ at these orders?