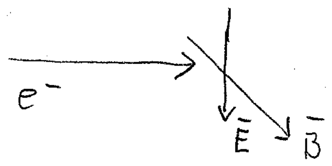
## 1.11 Historical developments: (11-50 in Griffiths)

- Thomson 1897: $\underline{e}^-$ charge/mass ratio

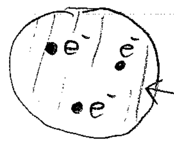



$$\bar{F} = -e(\bar{E} + \bar{v} \times \bar{B})$$

$\Rightarrow$ get $v$ $\Rightarrow$ use $\bar{F} = m\bar{a}$

$\Rightarrow$ get $\frac{e}{m}$

Atom:

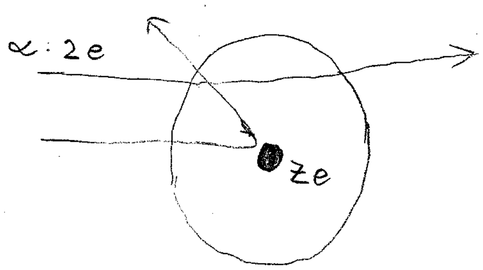



positive background charge

- Rutherford 1911:

  $\alpha$-particles scatter from gold atoms:

  $\Rightarrow$ Tiny nucleus which contains all +-charge and $\sim$ all of mass




$\Rightarrow$ much more large-angle scatterings than predicted by smooth background

- Chadwick 1932: neutron, n. Nuclei: p and n.
- Apparently complete picture of matter! But what force binds p's and n's together?
- 1934 Yukawa proposed a nuclear interaction, with new intermediate particle: "pion"
- (1937)
- 1946-47: <u>myons</u> and <u>pions</u> found in cosmic rays. (Powell) ($\mu$: "unnecessary" particles exist!)

- Dirac equation for relativistic electron (1927)
  $\Rightarrow$ predicts antielectron ? Could it be proton ? No!
- Anderson 1932: positive particle with mass of e found in cosmic ray experiment $\rightarrow$ positron
  $\Rightarrow \exists$ antimatter!

- 1930: missing energy in $\beta$-decay:
  $${}^{64}_{29}Cu \rightarrow {}^{64}_{30}Zn + e^- \quad ; \quad E_{Cu} > E_{Zn} + E_{e^-}$$
  (and, above all, $e^-$ spectrum very wide)
  W. Pauli: 3rd particle, neutrino (actually, anti-)
- 53-56: $\bar{\nu}$ seen directly: (Cowan + Reines)
  $$\bar{\nu} + p \rightarrow n + e^-$$
  ($\bar{\nu}$ from nuclear reactor)

~1950 QED developed (Feynman, Schwinger, Tomonaga, Dyson)
$\rightarrow$ 1st quantum field theory, fantastic success!

QCD established
- 1964: Quark model (Gell-Mann, Zweig) - bookkeeping device
- End of 60's: quarks "seen" within protons,
- 1973: QCD, theory of strong interactions (Politzer, Gross, Wilczek, asymptotic freedom)
- 1974: c found ($J/\psi$, $c\bar{c}$)

EW
- ~1965: Electroweak theory (Glashow, Weinberg, Salam) (predicts W, Z)
- 1983: found at LEP at CERN $\rightarrow$ S.M. established

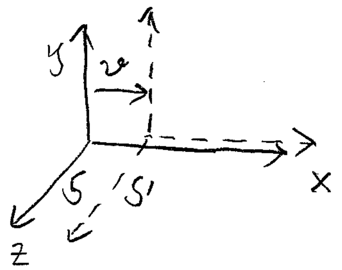
- 1975: $\tau$ found ; -81: b found $\rightarrow$ 3th generation
- 1995: t (top) found, Fermilab
- 2010 : Higgs ?

## 2. Relativistic kinematics

- Recall features from special relativity:

  2 inertial frames $S$, $S'$, and

  $S'$ moves with velocity $v$ wrt. $S$.




- Now let $\gamma = \dfrac{1}{\sqrt{1-v^2}} \geq 1$

$$\begin{cases} x' = \gamma(x - vt) \\ y' = y \\ z' = z \\ t' = \gamma(t - vx) \end{cases} \Leftrightarrow \begin{cases} x = \gamma(x' + vt') \\ y = y' \\ z = z' \\ t = \gamma(t' + vx') \end{cases}$$

Inverse: just switch $v \to -v$

$\Rightarrow$

1. Simultaneity:
   - 2 points $A, B$, with $t_A = t_B$.

   $\Rightarrow t'_A = t'_B + \gamma v (x_B - x_A)$ Not simultaneous

2. Lorentz contraction $\rightarrow$ compression by $1/\gamma$
   - Stick at rest in $S'$, endpoints at $x' = 0, L'$

   $\Rightarrow$ in $S$, endpoints move as

   $$\begin{cases} x_1 = \gamma v t'_1 \\ t_1 = \gamma t'_1 \end{cases} \qquad \begin{cases} x_2 = \gamma(L' + vt'_2) \\ t_2 = \gamma(t'_2 + vL') \end{cases}$$

   Set $t_1 = t_2 \Rightarrow$

   $$\underline{L} \equiv x_2 - x_1 = -\gamma v t'_1 + \gamma L' + \gamma v(t'_1 - vL') = \underline{\frac{L'}{\gamma}}.$$

3. Time dilatation $\rightarrow$ time slowdown by $\gamma$
   - Clock at origin of $S'$, ticks intervals $T'$: points $(0,0)_A$, $(0,T')_B$

   $\Rightarrow \underline{T} = t_B - t_A = \underline{\gamma T'}$

- **Four-vectors** ("nelivektori")

  - Coordinate:

$$x^\mu : \quad x^0 = t, \quad x^1 = x, \quad x^2 = y, \quad x^3 = z$$

$$x^\mu = (t, \vec{x})$$

  - Four-momentum:

$$p^\mu : \quad p^0 = E, \quad p^1 = p_x, \quad p^2 = p_y, \quad p^3 = p_z$$

$$p^\mu = (E, \vec{p}) \qquad \left( \left(\tfrac{E}{c}, \bar{p}\right) \text{ if } c \neq 1 \right)$$

  - Four-velocity: (proper velocity)

$$\boxed{U^\mu = \frac{dx^\mu}{d\tau}}$$

$dx^\mu$ ← coordinate of particle in lab frame

$d\tau$ ← proper time of particle = time in rest frame of particle

"Normal" velocity in lab frame

$$\vec{v} = \frac{d\vec{x}}{dt} \Rightarrow \text{"}V^\mu\text{"} = \frac{dx^\mu}{dt} = (1, \vec{v}) \qquad ((c, \vec{v}))$$

time dilatation: $\underline{dt = \gamma\, d\tau} \Rightarrow$

$$\boxed{U^\mu = \gamma(1, \vec{v})} = \text{"}\gamma V^\mu\text{"}$$

(Why $U^\mu$ and not $V^\mu$? $V^\mu$ does not transform as a proper 4-vector in Lorentz-transformations!)

- For a massive particle: $\boxed{p^\mu = m U^\mu}$
- $p^\mu$ is <u>always</u> conserved!
- Differential operators:

$$\partial_\mu \equiv \frac{\partial}{\partial x^\mu} \quad ; \quad \partial^\mu \equiv \frac{\partial}{\partial x_\mu}$$

- **Metric tensor**

$$g_{\mu\nu} = g^{\mu\nu} = \begin{array}{c|cccc} & 0 & & & \\ 0 & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \end{array} = \mathrm{diag}(1,-1,-1,-1)$$

(standard convention in particle physics)

Lowers/raises indices:

$$A_\mu = g_{\mu\nu} A^\nu \equiv \sum_{\nu=0}^{3} g_{\mu\nu} A^\nu \quad \text{(Einstein sum convention)}$$

repeated, sum over

Thus, $g_\mu{}^\nu = g_{\mu\alpha} g^{\alpha\nu} = \delta_\mu{}^\nu = \mathrm{diag}(1,1,1,1)$
$= g^\mu{}_\nu$

- Notation: $x^\mu$: any of $x^{0..3}$; $x^i$: any of $x^{1..3}$
  Greek, roman

- **Scalar product** **(dot product)**

  - $\underline{A \cdot B} = A_\mu B^\mu = g_{\mu\nu} A^\mu B^\nu = A^0 B^0 - A^1 B^1 - A^2 B^2 - A^3 B^3$
  $= \underline{A^0 B^0 - \vec{A} \cdot \vec{B}}$

  - $A^2 = A \cdot A = A^{02} - \vec{A} \cdot \vec{A}$

  - $\Box = \partial^\mu \partial_\mu = \partial_0^2 - \partial_i \partial_i = \frac{\partial^2}{\partial t^2} - \vec{\nabla}^2$

$$\boxed{\text{Scalar products always invariant in Lorentz-transformations!}}$$

("$v_\mu v^\mu$" is <u>not</u>, $U_\mu U^\mu$ is)

- Useful relations:

$$E^2 = \vec{p}^2 + m^2 \quad \Leftrightarrow \quad p^2 = m^2$$

$$\left.\begin{array}{l} p = mu = m\gamma(1, \vec{v}) \\ \Rightarrow E = m\gamma, \quad \vec{p} = m\gamma\vec{v} \end{array}\right\} \quad m \neq 0$$

$$\Rightarrow \underline{\vec{p} = \vec{v} E} \qquad (\text{any } m)$$

# Lorentz group

- Transformations of four-vectors which preserve scalar products: $\Lambda^\mu{}_\nu$

- Lorentz-transformation $\underline{A'^\mu = \Lambda^\mu{}_\nu A^\nu}$

- Invariance:

$$A' \cdot B' = g_{\mu\nu} A'^\mu B'^\nu = g_{\mu\nu} \Lambda^\mu{}_\alpha \Lambda^\nu{}_\beta A^\alpha A^\beta$$
$$= g_{\alpha\beta} A^\alpha A^\beta$$

Must be true for any $A, B \Rightarrow$

$$\boxed{g_{\mu\nu} \Lambda^\mu{}_\alpha \Lambda^\nu{}_\beta = g_{\alpha\beta}} \qquad \Lambda^\mu{}_\alpha = (\Lambda^T)_\alpha{}^\mu$$

$$\Rightarrow \underline{\Lambda^T g \Lambda = g} \Rightarrow \underline{\det \Lambda = \pm 1}$$

and $(\alpha = \beta = 0)$

$$(\Lambda^0{}_0)^2 - \sum_{i=1}^{3} (\Lambda^i{}_0)^2 = 1 \Rightarrow (\Lambda^0{}_0)^2 \geq 1$$
$$\Rightarrow \underline{\Lambda^0{}_0 \geq 1 \vee \Lambda^0{}_0 \leq -1}$$

- <u>Proper</u> Lorentz transformations : $\underline{\Lambda^0{}_0 \geq 1}$, $\underline{\det \Lambda = 1}$

(boosts, rotations)

- Examples:
  - Rotation $\vec{x}' = S\vec{x}$ ; $S^T = S^{-1}$

$$\Lambda = \left(\begin{array}{c|c} 1 & 0 \\ \hline 0 & S \end{array}\right) \qquad \det \Lambda = 1, \quad \Lambda^0{}_0 = 1$$

  - Boost along x-axis:

$$\Lambda = \begin{bmatrix} \gamma & -\gamma v & 0 & 0 \\ -\gamma v & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\det \Lambda = 1$, $\Lambda^0{}_0 > 1$

- Spatial reflection $\Lambda = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$ $x_0 \to x_0$, $\vec{x} \to -\vec{x}$

- Time reversal: $\Lambda = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$ Det $= -1$

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- Notations:

  $a_\mu$ covariant Lorentz-vector

  $a^\mu$ contravariant

$$\begin{cases} a^2 > 0 & \text{timelike} \quad ((1,\bar{0})) \\ a^2 = 0 & \text{lightlike} \\ a^2 < 0 & \text{spacelike} \quad ((0,\bar{a})) \end{cases}$$

⌈Name comes from the fact

$$p^2 = E^2 - \bar{p}^2 = m^2$$

Thus, for light (or for any particle with $m=0$) $p^2 = 0$. ($p^2 < 0$: tachyons)

For massive particles $p^2$ timelike. ⌋

Proper Lorentz transformations

keep timelike vectors timelike, spacelike spacelike (and lightlike lightlike)

# 3. Symmetries

- Have a central role in particle physics (and physics in general...)

- **Symmetry** of nature (Hamiltonian, action) $\Rightarrow$ **Conservation law** (**Noether's theorem**)

- Example: time evolution operator in Q.M.

$$\underline{U(t'-t) = e^{-i\hat{H}(t'-t)}}$$

$$\Rightarrow i\partial_t |\psi\rangle_s = \hat{H}|\psi\rangle \Rightarrow |\psi(t)\rangle_s = e^{-i\hat{H}t}|\psi(0)\rangle_s = U(t)|\psi(0)\rangle_s$$

in Schrödinger picture.

- If we have operator $\hat{A}$ which commutes with $\hat{H}$:

$$\underline{[\hat{A},\hat{H}] = 0}$$

$$\Rightarrow \langle\hat{A}\rangle(t) \equiv \langle\psi(t)|\hat{A}|\psi(t)\rangle = \langle\psi(0)|\hat{A}|\psi(0)\rangle = \langle\hat{A}\rangle(0)$$

Expectation value of $\hat{A}$ is conserved!

| Symmetry | Conservation law | "operator" |
|---|---|---|
| time translation | Energy | $i\partial_t = \hat{H}$ |
| space transl. | momentum | $\hat{p} = -i\vec{\nabla}$ |
| rotation | angular momentum | $\hat{L}$, angular mom. op. |
| local phase: $e^{i\theta(x)}\psi(x)$ (gauge symmetry) | (Electric) charge | |
| "Internal" symmetry (e.g. baryon number) | Quantum number (B) | |

Symmetry operations form a group: (G)

1. If $R_1 \in G$, and $R_2 \in G \Rightarrow R_1 R_2 \in G$
2. $\exists\, \underline{I} \in G$ so that $IR = RI = R$ for all $R \in G$
3. For all $R \in G$ there is inverse $R^{-1} \in G$ so that $RR^{-1} = R^{-1}R = I$
4. Associativity: $R_1(R_2R_3) = (R_1R_2)R_3$

For Abelian groups, $R_1R_2 = R_2R_1$, non-Abelian $\neq$.

Common continuous groups in physics:

| | | | | |
|---|---|---|---|---|
| $U(N)$ | : $N\times N$ | unitary matrices, | $U^\dagger = U^{-1}$ | |
| $SU(N)$ | : | " | $U^\dagger = U^{-1}$, | $\det U = 1$ |
| $O(N)$ | : $N\times N$ | orthogonal matrices, | $O^T = O^{-1}$ | |
| $SO(N)$ | : | " | $O^T = O^{-1}$, | $\det O = 1$ |

- 3d rotations: $\vec{x}\,' = S\vec{x}$, $S \in \underline{SO(3)}$
  - allow reflections too $x_i \to -x_i \Rightarrow S \in \underline{O(3)}$
- Complex rotations: $\underline{z' = Uz}$, $z, z' \in \mathbb{C}$, $\|z'\| = \|z\| \Rightarrow U \in \underline{U(1)}$

- Discrete groups: finite number of elements.
  E.g. rotations of a square: 4 elements, rotation by 0°, 90°, 180°, 270°

## 3.2 Representations

- Every group can be represented by matrices: For every $A \in G$ there exists matrix $M_A$, so that if $AB = C \Rightarrow M_A M_B = M_C$
- Simplest, trivial representation: $M_A = I$ for all $A$.
- Faithful representation: if $A \to M_A$ and $B \Rightarrow M_B$, $M_A = M_B$ if and only if $A = B$. (Bijection)
- Fundamental representation for a group of matrices is it itself: for example, SU(N) fundamental representation is $N \times N$ special unitary matrices (det=1)
- Reducible rep: (block diagonal) $M_A = \begin{pmatrix} M_A^1 & 0 \\ 0 & M_A^2 \end{pmatrix}$ for all $A \in G$

  $M_A = M_A^1 \oplus M_A^2$
- Irreducible rep: not reducible.
- For example, SU(2) has irred. representations of all integer dimensions,

  $1, \; 2, \; 3, \; 4, \; \ldots$

  $1$: trivial; $2$: fundamental
- New reps can be formed by taking direct products, e.g. SU(2)

  $$\underbrace{2 \otimes 2}_{4\times4\text{-matrix}} = 1 \oplus 3$$

  4×4-matrix, $M^{(2\otimes2)}_{ab} = M^{(2)}_{\alpha\beta} \times M^{(2)}_{\gamma\delta}$

  where $a = \alpha + 2(\gamma - 1)$; $b = \beta + 2(\delta - 1)$

  e.g.

| $a$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $(\alpha\beta)$ | (11) | (21) | (12) | (22) |