1. **Honeycomb lattice**
   (a) Verify that the construction

$$\mathbf{a}_1 = a\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right), \qquad \mathbf{a}_2 = a\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$$

$$\mathbf{v}_1 = a\left(\frac{1}{2\sqrt{3}}, 0\right), \qquad \mathbf{v}_2 = a\left(-\frac{1}{2\sqrt{3}}, 0\right),$$

   is correct so that the distance between all neighboring points is identical.
   (b) Sketch the neighborhoods of two particles in the honeycomb lattice which are not identical, and describe the rotation that would be needed to make them identical.

2. **Hexagonal lattice**
   The hexagonal lattice may be viewed as a special case of the centered rectangular lattice. Find the ratio between the height and the width of the rectangle for which the centered rectangular lattice would become hexagonal.

3. **Allowed symmetry axes**
   Consider two-dimensional Bravais lattice.
   (a) Try to justify the following: if the lattice is left invariant after rotation around an arbitrary point in the plane, it is left invariant after rotation around every lattice point.
   (b) Consider two nearest-neighbor points in the lattice and choose the primitive vector $\mathbf{a}_1$ to be between them. List the alternatives we have for choosing $\mathbf{a}_2$.
   (c) Assume that the lattice is left invariant after rotation by angle θ. Construct $\mathbf{a}_2$ from $\mathbf{a}_1$ using this rotation. Find the condition that the image of $\mathbf{a}_1$ under rotation by $-\theta$ is a lattice point. Using this, show that the only allowed axes are one-, two-, three-, four-, and sixfold.

4. Hexagonal close–packed lattice
   a) Verify that the basis vectors

$$\mathbf{a}_1 = (a, 0, 0)\ , \quad \mathbf{a}_2 = \left(\tfrac{a}{2}, \tfrac{a\sqrt{3}}{2}, 0\right)\ , \quad \mathbf{a}_3 = (0, 0, c)\ ,$$
$$\mathbf{v}_1 = (0, 0, 0)\ , \quad \mathbf{v}_2 = \left(\tfrac{a}{2}, \tfrac{a}{2\sqrt{3}}, \tfrac{c}{2}\right)$$

   result in placing atoms directly over the centers of triangles along the c axis.
   b) Verify that the hcp lattice forms a close–packed structure when $c/a = \sqrt{8/3}$.