

1. **Relation between the volumes of the unit cells**

Show that the volume of the unit cell in the reciprocal lattice is $(2\pi)^3/v$, where v is the volume of the unit cell in the direct lattice.

2. **Density of states and electrons**

Find the density of k and energy states for an ideal noninteracting Fermi gas in three dimensions. Also find a relation between the Fermi wave vector k_F ($\varepsilon_F = \hbar^2 k_F^2 / 2m$) and the density of electrons.

3. **Fermi energy and wave vector of copper**

Calculate the Fermi energy ε_F and the Fermi wave vector k_F of copper treating its conduction electrons as an ideal noninteracting Fermi gas. The copper has the fcc structure with $a = 3.61 \text{ \AA}$, and one conduction electron per atom.

4. **van Hove singularities**

- a) Consider one-dimensional crystal and a value of $\vec{k}$ near which $\mathcal{E}_{n\vec{k}} \approx \mathcal{E}_{\max} - k^2$. Find the singularity that will be produced in $g(\varepsilon)$.
- b) Repeat this for a three-dimensional crystal.