1. **Cyclotron effective mass**
For free electron $\mathcal{E}(\mathbf{k}) = \hbar^2 k^2/2m$. Calculate $\partial A(\mathcal{E}, k_z)/\partial \mathcal{E}$ and show that the general expression for the period in a magnetic field reduces to the free electron result.

2. **Effective mass tensor**
For electrons near a band minimum (or maximum) $\mathcal{E}(\mathbf{k})$ has the form

$$\mathcal{E}(\mathbf{k}) = \text{constant } + \frac{\hbar^2}{2}(\mathbf{k} - \mathbf{k}_0) \cdot \mathbf{M}^{-1} \cdot (\mathbf{k} - \mathbf{k}_0),$$

where the matrix $\mathbf{M}$ is independent of $\mathbf{k}$. Calculate the cyclotron effective mass using $m^* = \frac{\hbar^2}{2\pi} \frac{\partial A(\mathcal{E}, k_z)}{\partial \mathcal{E}}$ and show that it is independent of $\mathcal{E}$ and k_z and given by

$$m^* = \left(\frac{|\mathbf{M}|}{\mathbf{M}_{zz}} \right)^{1/2},$$

where $|\mathbf{M}|$ is the determinant of the matrix $\mathbf{M}$.

3. **Effective mass tensor**
When $\mathcal{E}(\mathbf{k})$ is as in problem 2, the semiclassical equations of motion are linear, and therefore easily solved.
   a) Assuming that the mean free time between electron collisions is τ, show that the DC conductivity is given by

$$\sigma = ne^2 \tau \mathbf{M}^{-1}$$

   b) Rederive the result of the problem 2 by finding explicitly the time dependent solutions to

$$\mathbf{M} \cdot \frac{d\mathbf{v}}{dt} = -e \left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{H} \right),$$

   and noting that the angular frequency is related to m^* by $\omega = eH/m^*c$.