1. **DC conductivity**
Try to argue why the electric field is in parallel with the induced current in the case of cubic symmetry. How does the situation change in the case of hexagonal close-packed structure?

2. **AC conductivity**
The response of the conduction electrons to an electric field

$$\mathbf{E}(\mathbf{r},t) = \mathrm{Re}[\mathbf{E}(\mathbf{q},\omega)e^{i(\mathbf{q}\cdot\mathbf{r}-\omega t)}],$$

which depends on position as well as time, requires some special consideration. Such a field will in general induce a spatially variying charge density

$$\rho(\mathbf{r},t) = -e\delta n(\mathbf{r},t),$$

$$\delta n(\mathbf{r},t) = \mathrm{Re}[\delta n(\mathbf{q},\omega)e^{i(\mathbf{q}\cdot\mathbf{r}-\omega t)}].$$

Since electrons are conserved in collisions, the local distribution appearing in the relaxation-time approximation must correspond to a density equal to the actual instantaneous local density $n(\mathbf{r},t)$. Thus even at uniform temperature one must allow for a local chemical potential of the form

$$\mu(\mathbf{r},t) = \mu + \delta\mu(\mathbf{r},t),$$

$$\delta\mu(\mathbf{r},t) = \mathrm{Re}[\delta\mu(\mathbf{q},\omega)e^{i(\mathbf{q}\cdot\mathbf{r}-\omega t)}],$$

where $\delta\mu(\mathbf{q},\omega)$ is chosen to satisfy (to linear order in $\mathbf{E}$) the condition

$$\delta n(\mathbf{q},\omega) = \frac{\partial n_{\mathrm{eq}}(\mu)}{\partial\mu}\delta\mu(\mathbf{q},\omega).$$

a) Show, as a result, that at uniform temperature the equilibrium distribution is now

$$g(\mathbf{r},\mathbf{k},t) = g_0 + \mathrm{Re}[\delta g(\mathbf{q},\mathbf{k},\omega)e^{i(\mathbf{q}\cdot\mathbf{r}-\omega t)}],$$

$$\delta g(\mathbf{q},\mathbf{k},\omega) = \left(-\frac{\partial f}{\partial\mathcal{E}}\right)\frac{\delta\mu(\mathbf{q},\omega)/\tau - e\mathbf{v}(\mathbf{k})\cdot\mathbf{E}(\mathbf{q},\omega)}{1/\tau - i[\omega-\mathbf{q}\cdot\mathbf{v}(\mathbf{k})]}.$$

b) By constructing the induced current and charge densities from the distribution function, show that the choice of $\delta\mu(\mathbf{q},\omega)$ is precisely what is required to insure that the equation of continuity (local charge conservation)

$$\mathbf{q}\cdot\mathbf{j}(\mathbf{q},\omega) = \omega\rho(\mathbf{q},\omega) \quad \left(\nabla\cdot\mathbf{j} + \frac{\partial\rho}{\partial t} = 0\right),$$

is satisfied.

3. **Thermal conductivity**
Consider a metal in which thermal and electric currents flow simultaneously. The rate at which heat is generated in a unit volume is related to the local energy and number densities by

$$\frac{dq}{dt} = \frac{du}{dt} - \mu \frac{dn}{dt},$$

where μ is the local chemical potential. Using the equation of continuity,

$$\frac{dn}{dt} = -\nabla \cdot \mathbf{j}^n,$$

and the fact that the rate of change of the local energy density is determined by the rate at which electrons carry energy into the volume plus the rate at which the electric field does work,

$$\frac{du}{dt} = -\nabla \cdot \mathbf{j}^{\mathcal{E}} + \mathbf{E} \cdot \mathbf{j},$$

show that the heat equation can be written in the form

$$\frac{dq}{dt} = -\nabla \cdot \mathbf{j}^q + \mathcal{E} \cdot \mathbf{j},$$

where $\mathbf{j}^q$ is the thermal current (defined in the lecture notes) and $\mathcal{E} = \mathbf{E} + (1/e)\nabla\mu$.