1. Calculate

$$\text{a)}\ \nabla\left(\frac{1}{r^3}\right) \qquad \text{b)}\ \nabla\cdot\left(\nabla\frac{1}{r}\right) \qquad \text{c)}\ \nabla\left(\frac{x-y}{r^n}\right)$$

2. Calculate the integral

$$\int_{\mathbb{R}^3} dV \frac{e^{-ar}}{r}$$

where dV is the three-dimensional volume element and $a \in \mathbb{R}$ is a constant.

3. Diffraction from a single slit. Derive the intensity distribution

$$I = I_0 \frac{\sin^2\beta}{\beta^2}$$

of the interference pattern. Here $\beta = \frac{1}{2}kb\sin\theta$, b is the width of the slit and k is the wave number of the radiation field. What happens in the limit $kb \to 0$?
You may find the Huygen's principle useful. For a detailed discussion on the single slit diffraction, see e.g. Introduction to Optics, Pedrotti and Pedrotti, second edition.

4. In the double-slit experiment, there are two slits (width b), whose corresponding points are separated by a distance a. The intensity distribution is

$$I = 4I_0 \frac{\sin^2\beta}{\beta^2}\cos^2\alpha$$

where β is the same quantity as in problem 1.3 and $\alpha = \frac{1}{2}ka\sin\theta$. Derive this formula.
For a detailed discussion on the double slit diffraction, see reference given in problem 1.3.

5. Solve the differential equation

$$y''(x) - 4y'(x) + 2y(x) = \sin x.$$

See e.g. Fundamentals of Differential Equations and Boundary Value Problems, Nagle, second edition, for a discussion on how to solve differential equations of this form.