

1. Find the simultaneous eigenfunctions of the Hamiltonian

$$H = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial \phi^2}$$

and the angular momentum operator L_z .

2. Sketch the probability distributions of the quantum ring Hamiltonian eigenfunctions

$$\begin{aligned} \text{(i)} \quad \psi_3(\phi) &= \cos 3\phi, \\ \text{(ii)} \quad \psi_4(\phi) &= \cos 4\phi. \end{aligned}$$

Use the orbital representation introduced in the lectures.

3. Construct an explicit expression for the wave packet

$$\psi(\phi) = N \sum_{m_I=0}^{\infty} \frac{1}{m_I!} e^{im_I\phi}.$$

Determine N so that the wave packet is normalized to unity (employ the result $\int_0^{2\pi} e^{2\cos\phi} d\phi \approx 2\pi \times 2.280$). Sketch the form of $|\psi(\phi)|^2$ for $0 \leq \phi \leq 2\pi$.

4. Calculate $\langle \phi \rangle$, $\langle \sin \phi \rangle$ and $\langle L_z \rangle$ for the wave packet of the previous problem. You may need the integral $\int_0^{2\pi} \cos \phi e^{2\cos \phi} d\phi \approx 2\pi \times 1.591$.
5. Show that $[\mathbf{L}, r^2] = 0$ and $[H, \mathbf{L}] = 0$ where $r^2 = x^2 + y^2 + z^2$ and the potential energy part of the Hamiltonian depends only on r (as opposed to $\mathbf{r}$).