1. Let us assume that a hydrogen atom is in a 3p state. Show that the radial part of its wave function is

$$u_{31}(r) = \frac{4}{81\sqrt{6}}\, e^{-\frac{r}{3}}\, r^2(6-r).$$

2. Derive the dipole selection rules $\Delta l = \pm 1$, $\Delta m = \pm 1$ by calculating the matrix elements of the x and y components of the dipole operator $\mathbf{D} = e\mathbf{r}$. You may need the identity $(2\ell+1)\sqrt{1-t^2}P_\ell^{m-1}(t) = P_{\ell+1}^m(t) - P_{\ell-1}^m(t)$.

3. Consider a one-dimensional anharmonic oscillator. Let the perturbation be of the form $H_1 = \sigma\hbar\omega x^3$. Calculate the matrix elements

$$\begin{aligned}
\langle\psi_{n+3}|x^3|\psi_n\rangle &= \sqrt{\frac{(n+3)(n+2)(n+1)}{8}}, \\
\langle\psi_{n+1}|x^3|\psi_n\rangle &= 3\left(\frac{n+1}{2}\right)^{\frac{3}{2}}, \\
\langle\psi_{n-1}|x^3|\psi_n\rangle &= 3\left(\frac{n}{2}\right)^{\frac{3}{2}}, \\
\langle\psi_{n-3}|x^3|\psi_n\rangle &= \sqrt{\frac{n(n-1)(n-2)}{8}}.
\end{aligned}$$

4. Consider an anharmonic oscillator in which the perturbation is of the form $H_1 = \sigma\hbar\omega x^3$. First, show that the energy levels are (to the second order in σ)

$$E_n = (n+\frac{1}{2})\hbar\omega - \sigma^2\left[\frac{15}{4}(n+\frac{1}{2})^2 + \frac{7}{16}\right]\hbar\omega.$$

Then, show that the energy difference

$$E_n - E_{n-1} = \left(1 - \frac{15}{2}\sigma^2 n\right)\hbar\omega.$$

Finally, determine the energy eigenstates to the first order in σ.
See the lecture notes section 17.2.3.

5. Calculate the van der Waals constant

$$
\begin{aligned}
C &= \frac{e^4}{2E_{1s}} \langle 1s; 1s | (x_A x_B + y_A y_B - 2 z_A z_B)^2 | 1s; 1s \rangle \\
&= \frac{e^4}{2E_{1s}} 6 \left| \langle 1s | \frac{r_A^2}{3} | 1s \rangle \right|^2 \\
&= 6 e^2 a_0^5.
\end{aligned}
$$