1. The wave function of a particle in one dimension is
$$\Psi(x,t) = xe^{-2|x|-i\omega t}.$$
a) Calculate the probability density.
b) Let us consider intervals of the form $[x-\frac{1}{4},\, x+\frac{1}{4}]$, where $x \in \mathbb{R}$. Determine the interval in which the particle is most likely found.
c) What is the probability to find the particle at range $[-2, 2]$?
Part b) is the most challenging.

2. Particle is described by a wave function
$$\Psi(r) = \frac{e^{(ik-a)r}}{r}; \qquad a > 0 \text{ constant.}$$
Calculate the probability current density $\mathbf{S}$, when $r^2 = x^2 + y^2 + z^2$. How does $\mathbf{S}$ behave for large values of r?

3. A particle is described by the wave function
$$\Psi(\mathbf{r},t) = \frac{1}{N} e^{-2br} e^{-i\omega t}; \qquad b > 0 \text{ constant.}$$
Calculate the normalization factor N so that
$$\int |\Psi(\mathbf{r},t)|^2 \, dV = 1.$$
Then calculate the expectation values $\langle \mathbf{p} \rangle$ and $\langle E \rangle$.
The momentum expectation value $\langle \mathbf{p} \rangle$ vanishes.

4. A particle moves in a potential $V = V(x)$. Show that the quantum mechanical expectation values satisfy
$$\frac{d}{dt}\langle p_x \rangle = -\left\langle \frac{\partial V}{\partial x} \right\rangle$$
where $p_x = -i\hbar\frac{\partial}{\partial x}$ is the momentum operator in the x-direction.

5. Show that
$$\left\langle \frac{\partial V}{\partial x} \right\rangle = \frac{\partial V}{\partial x}\Big|_{x=\langle x \rangle}$$
when $V(x) = ax^2$. Does the above equation hold for $V(x) = bx^3$?