1. Show that the operators
   a) $p = -i\hbar\nabla$
   b) $H = -\frac{\hbar^2}{2m}\nabla^2 + V$, where V is a real function
   c) x, the position operator,
   are Hermitian.

2. Show that
$$\int_{-\infty}^{\infty} \frac{\sin gx}{x} dx = \pi, \quad \text{when } g > 0.$$
   In one version of the solution, one calculates a certain contour integral of the function $f(z) = e^{igz}/z$ and employs the Cauchy's integral theorem.

3. Show that a linear combination of two square-integrable, complex-valued functions is also square-integrable.

4. a) Calculate the density $\rho(k_F)$ of the one-dimensional Fermi gas as a function of the Fermi wave vector k_F.
   b) Calculate the density of states $g(E)$ as a function of energy E in one and two dimensions.

5. Assume that the Hamiltonian of the system, H, has only two eigenstates, for which $H|\psi_i\rangle = \hbar\omega_i|\psi_i\rangle$, where $i \in \{1, 2\}$. Furthermore, let us assume that the system is in a superposition state $|\Psi\rangle = c_1|\psi_1\rangle + c_2|\psi_2\rangle$. We want to measure a dynamical quantity β to which corresponds a hermitian operator B. The operator B satisfies the eigenvalue equation $B|u_j\rangle = b_j|u_j\rangle$, where $j \in \{1, 2\}$ and $b_1 \neq b_2$.
   a) What are the possible results of the measurement?
   b) What are the probabilities to get these results?