1. A particle moves in a potential $V = V(x)$. Show that the quantum mechanical expectation values satisfy

$$\frac{d}{dt}\langle p_x \rangle = -\left\langle \frac{\partial V}{\partial x} \right\rangle$$

where $p_x = -i\hbar\frac{\partial}{\partial x}$ is the momentum operator in the x-direction.
Like the proof for the result $\frac{d}{dt}\langle \mathbf{r} \rangle = \frac{\langle \hat{\mathbf{p}} \rangle}{m}$ *in the lecture notes, also this proof relies on an assumption that a certain surface integral vanishes.*

2. Let us consider a particle in an infinite potential well. Calculate the expectation value of its
   a) kinetic energy,
   b) potential energy.

3. Let us consider the eigenvalue equation

$$\hat{A}u = \lambda u, \qquad \hat{A} = -\frac{d^2}{dx^2},$$

under the boundary condition $u(-a) = u(a) = 0$ (a is a positive real number). Determine those eigenfunctions u that correspond to a positive eigenvalue λ .

4. Particles scatter from a potential step. Show that from the conservation of the probability current it follows that the reflection and transmission coefficients R and T satisfy the condition

$$R + T = 1.$$

5. A particle scatters from a potential step of height V_0. The energy of the particle $E < V_0$.
   a) Show that the probability current inside the potential step vanishes.
   b) Show that total reflection happens.
   c) Estimate the penetration depth by evaluating $\langle x \rangle$ inside the step.
   d) Show that there is a phase shift between the incoming and the scattered wave.
   e) Study the behavior of the probability density outside the potential step.