1. Calculate the first three Hermite polynomials by using the generating function. After that, calculate the third Hermite polynomial again by using the recurrence relation.

2. Calculate the expectation values of position and momentum for the harmonic oscillator energy eigenstates.

3. Show that the uncertainty relation
$$\Delta x \Delta p = (n + \frac{1}{2})\hbar$$
holds for the harmonic oscillator energy eigenstates.

4. Let us consider the Hermite polynomials $H_5(\xi) = \sum_{k=0}^{2} \bar{a}_{2k}\xi^{2k+1}$ and $H_6(\xi) = \sum_{k=0}^{3} a_{2k}\xi^{2k}$. Calculate the ratios $\bar{a}_4 : \bar{a}_2 : \bar{a}_0$ and $a_6 : a_4 : a_2 : a_0$.

5. Show that the harmonic oscillator wave function satifies the recurrence relation
$$\alpha x \psi_n(x) = \sqrt{\frac{n+1}{2}}\psi_{n+1}(x) + \sqrt{\frac{n}{2}}\psi_{n-1}(x), \quad \alpha = \sqrt{\frac{m\omega}{\hbar}}.$$
Using this recurrence relation, calculate $\psi_3(x)$ assuming that you know $\psi_0(x)$ and $\psi_1(x)$.