1. Warm up

   a) Show that the eigenvalues of a Hermitian operator A are real and that the eigenkets of A corresponding to different eigenvalues are orthogonal.

   b) Show that if state ket $|\alpha\rangle = \sum_{a'} c_{a'}|a'\rangle$ is normalized then the expansion coefficients $c_{a'}$ must satisfy

$$\sum_{a'} |c_{a'}|^2 = 1.$$

2. Prove the Theorem 1 from lecture notes:

   If both of the basis $\{|a'\rangle\}$ and $\{|b'\rangle\}$ are orthonormalized and complete then there exists a unitary operator U so that

$$|b_1\rangle = U|a_1\rangle, |b_2\rangle = U|a_2\rangle, |b_3\rangle = U|a_3\rangle, \ldots$$

3. Consider the spin operators S_x, S_y and S_z in the $\{|S_z;\uparrow\rangle, |S_z;\downarrow\rangle\}$ basis.

   a) Write out the operators S_x, S_y and S_z in the $\{|S_z;\uparrow\rangle, |S_z;\downarrow\rangle\}$ basis.

   b) Compute the commutators $[S_x, S_y]$ and $[S^2, S_x]$ as well as the anticommutator $\{S_x, S_y\}$

   c) Let us define the ladder operators $S_\pm = S_x \pm iS_y$. Compute the $S_\pm|S_z;\uparrow\rangle$ and $S_\pm|S_z;\downarrow\rangle$.

4. Prove the Theorem 2 from lecture notes:

   If U is a unitary matrix, then the matrices X and $U^\dagger XU$ have the same trace and the same eigenvalues.

5. The translation operator for a finite (spatial) displacement is given by

$$\mathcal{T}(\mathbf{l}) = \exp\left(-\frac{i}{\hbar}\mathbf{p}\cdot\mathbf{l}\right)$$

where $\mathbf{p}$ and $\mathbf{l}$ are the momentum operator and displacement vector respectively.

a) Evaluate $[x_i, \mathcal{T}(\mathbf{l})]$

b) How does the expectation value $\langle x \rangle$ of position operator x change under translation?