1. Consider a orbital angular momentum state $|l = 2, m = 0\rangle$. Suppose this state is rotated by an angle β around the y axis. Find the probability for the new state to be found in states m=-2, -1, 0, 1, 2.

   You might found the definition of spherical harmonic function useful:

   $$Y_l^m(\theta, \phi) = e^{im\phi} \sqrt{\frac{(2l+1)(l-m)!}{4\pi(l+m)!}} P_l^m(\cos\theta)$$

   where the associated Legendre functions read

   $$P_l^m(x) = (-1)^m (1-x^2)^{m/2} \frac{d^m}{dx^m}(P_l(x))$$

   and the Legendre polynomials themselves

   $$P_k(x) = \frac{1}{2^k k!} \frac{d^k}{dx^k} \Big((x^2-1)^2 \Big).$$

2. Consider a system of two spin-$\frac{1}{2}$ particles. Derive the Clebsh Gordan coefficients , i.e. the transformation matrix from the uncoupled to the coupled base kets, in the case the orbital angular momentum is suppressed. Otherwise stated: couple the spin angular momentum of two spin-$\frac{1}{2}$ particles.

3. Consider spin-orbit coupling of an electron, in a state $l = 1$, $s = \frac{1}{2}$.

   a) Find the (simultaneous) eigenkets of $\mathbf{J^2}$, J_z, $\mathbf{S^2}$ and $\mathbf{L^2}$ by diagonalizing the $\mathbf{J^2}$ operator in the coupled basis.

   b) Find the matrix elements of the Thomas term $\mathbf{L}\cdot\mathbf{S}$, which is responsible for the fine structure of atomic spectral lines (we will come back to this later).

4. Find the general expression (Wigner formula) for the Euler rotation around the y axis.

$$
\begin{aligned}
d^{(j)}_{m'm}(\beta) &= \sum_k (-1)^{k-m+m'} \frac{\sqrt{(j+m)!(j-m)!(j+m')!(j-m')!}}{(j+m-k)!k!(j-k-m')!(k-m+m')!} \\
&\times \left(\cos\frac{\beta}{2}\right)^{2j-2k+m-m'} \left(\sin\frac{\beta}{2}\right)^{2k-m+m'} .
\end{aligned}
$$

Hint: We can consider the rotational properties of *any* quantum mechanical object with angular momentum j in terms of two uncoupled harmonic oscillators. In this case we can associate the number of spin-$\frac{1}{2}$ particles (n_1) in state spin up and the number of spin-$\frac{1}{2}$ particles (n_2) in state spin down with the angular momemtum quantum numbers (j, m) as

$$
j = \frac{n_1 + n_2}{2} \quad , \quad m = \frac{n_1 - n_2}{2}
$$

and implement the the formula

$$
|j, m\rangle = \frac{(a_1^\dagger)^{j+m}(a_2^\dagger)^{j-m}}{\sqrt{(j+m)!(j-m)!}}|0, 0\rangle .
$$

By applying the rotation operator on above identity will lead to the Wigner's formula.

5. Read the original papers on EPR paradox (Phys. Rev. 47,777 (1935)) and Bells "reply" (Physics, 1, 195 (1964)). The articles in pdf format can be found from the EXTRAS-section under titles EPR-paradox.pdf and Bell-reply.pdf. You will notice that the original EPR paper actually considers the x, p uncertainty relation (non-commuting operators in general) but the same argumentation would hold equally well for example for spin. By the time of Bell's paper the paradox had already found it's modern version (with Alice and Bob measuring spin) that we considered at lectures.