1. Calculate the three lowest energy levels and their degeneracies for the following systems

   a) Three non-interacting spin-$\frac{1}{2}$ particles in a box of length $L$.

   b) Four non-interacting spin-$\frac{1}{2}$ particles in a box of length $L$

   You can assume that the particles have the same mass and that they are distiquishable.

2. The Hamiltonian for a spin 1 system is given by

$$H = AS_z^2 + B(S_z^2 - S_y^2)$$

   Solve this problem exactly to find the normalized energy eigenstates and eigenvalues. Is this Hamiltonian invariant under time reversal? How do the normalized eigenstates you obtained transform under time-reversal? Notice that such spin-dependent Hamiltonians actually appear in crystal physics.

3. Rotations and angular momentum eigenstates under time-reversal.

   a) What is the time-reversed state corresponding to $\mathcal{D}(R)|j,m\rangle$?

   b) Using the properties of time reversal and rotations, prove

$$\mathcal{D}^{(j)*}_{m'm}(R) = (-1)^{m-m'}\mathcal{D}^{(j)}_{-m',-m}(R)$$

   c) Prove $\Theta|j,m\rangle = i^{2m}|j,-m\rangle$.