1. In the lectures we looked through the two state system under a sinusoidal time-depedent potential (aka harmonic perturbation) by "solving", i.e. by borrowing the exact solution from Rabi and fellows.

   For comparison, let us now consider a similar problem but in the framework of time-dependent perturbation theory (i.e. use Dyson series to solve this problem). Namely, let the unperturped Hamiltonian read

$$H_0 = \begin{pmatrix} E_1^{(0)} & 0 \\ 0 & E_2^{(0)} \end{pmatrix}$$

   and the perturbation

$$V(t) = \begin{pmatrix} 0 & \lambda \cos \omega t \\ \lambda \cos \omega t & 0 \end{pmatrix}$$

   in which $\lambda \in \Re_+$. Let the system be initially in the lower state $|1\rangle$ represented as

$$|1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad ; \quad |2\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} .$$

   Find the probability as a function of time for the system to be found in the other $|2\rangle$ state. Notice that $E_2^{(0)} - E_1^{(0)}$ must not be close to $\pm\hbar\omega$ or otherwise the perturbation expansion (at first order) breaks down. Verify this explicitely by considering the $E_2^{(0)} - E_1^{(0)} \approx \pm\hbar\omega$ case as well.

2. Consider a Hydrogen atom in a static external electric field (i.e. the Stark effect). In the lectures, the lowest excited states were considered ($n = 2$). Work out the same problem to the lowest non-vanishing order for the second (principal value) excited states, i.e. the $n = 3$ case. You can ignore the relativistic effects again (Lamb shift) and spin-orbit force. Solve the problem properly, i.e. find the energy shifts and the corresponding zeroth order eigenkets.

   Admittedly, this one is a nerves/wrists/balls breaker... Nevertheless, if you look it through on your own, then you probably understand the concept of perturbation theory, Hydrogen atom wave functions as well as many other topics we have considered before, at least, on a satisfactory level.

   Take advantage of the symmetry/selection rule considerations. Also, feel free to use any material/software to break the integrals.

   p.s. These 2 problems should be enough for this week. Do not worry, though, the forthcoming topics during this course will be less calculable and so we will have time to practise time-dependent perturbation theory at the next weeks exercises as well.