1. Understanding the concepts of gauge, gauge invariance, local and global symmetry and the connection between all these. "Gauge theories in particle physics" starts with a little bit more extensive and elementary level discussion (compared to many other books) on gauge invariance in QED. This chapter can be found from the EXTRAS section by name qed_gauge.pdf. Read it and try to understand what is going on...

2. Show that a free photon field is polarized in the transverse direction i.e. that only 2 out of the 4 components of the polarization vector can be chosen independent. Notice, unlike in the lectures, you need to consider relativistic vector field: $\mathbf{A} \rightarrow A^\mu$ = four component vector field, i.e. $\hat{\epsilon} \longrightarrow \hat{\epsilon}^\mu$

   Hint: start from a covariant vector field A^μ and make use of the freedom to choose the gauge twice.

3. Verify by explicit calculation that

   a) the Hamiltonian operator of a quantized radiation field

   $$H = \frac{1}{2}\int(\mathbf{B}\cdot\mathbf{B} + \mathbf{E}\cdot\mathbf{E})d^3x$$

   reduces to the

   $$H = \sum_{\mathbf{k}}\sum_{\alpha}(N_{\mathbf{k},\alpha} + \frac{1}{2})\hbar\omega$$

   b) momentum

   $$\mathbf{P} = \frac{1}{c}\int(\mathbf{E}\times\mathbf{B})d^3x$$

   reduces to the

   $$\mathbf{P} = \sum_{\mathbf{k}}\sum_{\alpha}\hbar\mathbf{k}N_{\mathbf{k},\alpha}$$

4. The Klein-Gordon equation in a electromagnetic field reads

$$\frac{1}{c^2}[i\hbar\frac{\partial}{\partial t} - e\Psi]^2\psi(x,t) = \left([\frac{\hbar}{i}\nabla - \frac{e}{c}\mathbf{A}]^2 + mc^2\right)\psi(x,t).$$

Explore the non-relativistic limit of above.