1. "Show" that the Dirac equation describes a spin-$\frac{1}{2}$ particle.
   Hint: Consider constants of a motion (i.e. start from Heisenberg equation of motion) for a free spin $\frac{1}{2}$ particle and show that $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ alone does not commute with the Hamiltonian. Also, show that the generalized (to 4 dimensions) spin operator

   $$\Sigma_k = \begin{pmatrix} \sigma_k & 0 \\ 0 & \sigma_k \end{pmatrix} \quad ; \quad \mathbf{\Sigma} = (\Sigma_1, \Sigma_2, \Sigma_3)$$

   does not commute with the Hamiltonian either, but that the joint operator $\mathbf{J} = \mathbf{L} + \frac{1}{2}\mathbf{\Sigma}$ do commute with H.

2. Show that operator

   $$K = \beta\mathbf{\Sigma} \cdot \mathbf{J} - \beta\hbar/2 = \beta(\mathbf{\Sigma} \cdot \mathbf{L} + \hbar)$$

   commutes with the Hamiltonian (electron in a central potential)

   $$H = c\alpha \cdot \mathbf{p} + \beta mc^2 + V(r)$$

   Considering an electron in a central potential, operators H, K, $\mathbf{J}$ and J_3 form the maximal mutually commuting set of observables with eigenvalues E, $-\kappa$, $j(j+1)\hbar$ and $j_3\hbar$. In other words these observables label the Hydrogen atom quantum states within Dirac formalism.
   Show that κ and j are connected as

   $$\kappa = \pm(j + \frac{1}{2}).$$

   The sign of κ tells us, pictorially speaking, wether the electron spin is parallel ($\kappa < 0$) or antiparallel ($\kappa > 0$) to the total angluar momentum in the non-relativistic limit.

3. Finally we have all the tools in order to prove the spin-statistics connection that we needed to postulate in the framework of non-relativistic quantum mechanics. The prove is not trivial and thus instead of performing the calculations by yourself, check how Pauli did that. The paper can be found from the www by name EXTRAS/PhysRev58p716.pdf.