- IN THE CASE OF HARMONIC OSCILLATOR, WE FOUND THAT THE EXPECTATION VALUE OF $[X,P]$ COMMUTATOR WAS INDEPENDENT ON THE STATE $|n\rangle$
- THIS WAS DUE TO THE FACT THAT $[X,P] = i\hbar \mathbb{1}$ DOES NOTHING TO THE STATE $|n\rangle$
- HOWEVER, IN GENERAL $[A,B] = C$ ; ANOTHER OPERATOR AND

$\langle [A,B] \rangle$ <u>IS STATE DEPENDENT</u>

LET US LOOK AT SPIN

$$[S_x, S_y] = i\hbar S_z$$

LET US DENOTE $|S_z \uparrow\rangle \equiv |+\rangle$

$|S_z \downarrow\rangle \equiv |-\rangle$

THEN $\boxed{\text{STATE \#1} = |S_z\uparrow\rangle}$

$$\langle +|[S_x, S_y]|+\rangle = \langle +|i\hbar S_z|+\rangle$$

$$= i\hbar\langle +|\frac{\hbar}{2}|+\rangle = \frac{i\hbar^2}{2}\langle +|+\rangle$$

AND $\frac{1}{4}\left|\langle +|[S_x, S_y]|+\rangle\right|^2 = \frac{\hbar^4}{16}$

REMEMBER

$$|S_x\uparrow\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle + |-\rangle\right)$$

$\boxed{\text{STATE \#2} = |S_x\uparrow\rangle}$

AND

$$\frac{1}{4}\left|\langle S_x\uparrow|[S_x, S_y]|S_x\uparrow\rangle\right|^2$$

$$= \frac{1}{4}\left|\left(\langle +| + \langle -|\right)\frac{1}{\sqrt{2}}\,|i\hbar S_z|\,\frac{1}{\sqrt{2}}\left(|+\rangle + |-\rangle\right)\right|^2$$

$$= \frac{\hbar^2}{16} \left| \left( \langle +| + \langle -| \right) \frac{\hbar}{2} \left( |+\rangle - |-\rangle \right) \right|^2$$

$$= \frac{\hbar^4}{4 \cdot 16} \left| 1 - 0 + 0 - 1 \right|^2 = \underline{\underline{0}}$$

$\Rightarrow \langle \,|[S_x, S_y]|\, \rangle$ <u>DEPENDS ON THE STATE</u>

THE OTHER SIDE

$$S_x = \frac{\hbar}{2} \left( |+\rangle\langle -| + |-\rangle\langle +| \right)$$

AND

$$\langle + | S_x | + \rangle =$$

$$= \langle +| \left( |+\rangle\langle -| + |-\rangle\langle +| \right) |+\rangle \frac{\hbar}{2}$$

$$= \frac{\hbar}{2} \left( \langle -| + 0 \right) |+\rangle = 0$$

$$\langle + | S_x^2 | + \rangle = \langle +| \left\{ \left( |+\rangle\langle -| + |-\rangle\langle +| \right) \left( |+\rangle\langle -| + |-\rangle\langle +| \right) \right\} |+\rangle$$

$$= \langle + \Big[ |+\rangle\langle -\overset{=0}{|+\rangle}\langle -| + |+\rangle\langle -\overset{=1}{|-\rangle}\langle +| + |-\rangle\langle +\underset{=1}{|+\rangle}\langle -| + |-\rangle\langle +\underset{=0}{|-\rangle}\langle +| \Big] |+\rangle$$

$$= \langle +| \big( |+\rangle\langle +| + |-\rangle\langle -| \big) |+\rangle$$

$$= 1$$

$$\Rightarrow \langle + | S_x^2 | + \rangle = \frac{\hbar^2}{4}$$

$$\langle (\Delta S_x)^2 \rangle = \langle S_x^2 \rangle - \langle S_x \rangle^2$$

$$= \frac{\hbar^2}{4} - 0$$

SIMILARLY FOR $\langle (\Delta S_y)^2 \rangle = \frac{\hbar^2}{4}$

AND $\frac{\hbar^2}{4} \cdot \frac{\hbar^2}{4} \geq \frac{\hbar^4}{16}$ OK

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STATE #1 MINIMUM UNCERTAINTY

CALCULATE THE

$\langle (\Delta S_x)^2 \rangle \langle (\Delta S_y)^2 \rangle$ AT HOME

TO VERIFY : $0 \geq 0$ FOR STATE #2