

# Mathematical Methods.

## Problem set 11.

Hand-in 8/12-2008

Last problem set. Almost there!

1. By calculating

$$(Lu, v) = (u, L^*v)$$

find the adjoint operator  $L^*$  when

$$a) \quad Lu = u'' + a(x)u' + b(x)u, \quad u(0) = u'(1), \quad u(1) = u'(0),$$

$$b) \quad Lu = -(p(x)u')' + q(x)u, \quad u(0) = u(1), \quad u'(1) = u'(0).$$

The inner product is assumed to be

$$(u, v) = \int_0^1 v(x)u(x)dx.$$

2. For  $Lu = u''$ , for which sets of coefficients in the boundary conditions

$$a_1u(0) + b_1u(1) + c_1u'(0) + d_1u'(1) = 0$$

$$a_2u(0) + b_2u(1) + c_2u'(0) + d_2u'(1) = 0$$

is  $L$  self-adjoint?

3. Defining  $Lw$  from

$$(Lv, w) = (v, Lw)$$

find the meaning of  $Lw$  in the distribution sense, when

$$a) \quad Lu = u'', \quad u(0) = u'(0) = 0,$$

$$b) \quad Lu = u'', \quad u'(0) - \alpha u(0) = u'(1) - \beta u(1) = 0.$$

4. (Extra:) Solve the non-homogeneous boundary value problem

$$u'' - k^2u = f(x), \quad u(0) - u'(0) = a, \quad u(1) = b.$$